FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

NATURWISSENSCHAFTLICHE FAKULTÄT

CHAIR IN APPLIED ANALYSIS FAU

OBSERVER-BASED DATA ASSIMILATION IN TIME-DEPENDENT FLOWS IN GAS NETWORKS

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Mathematical Framework

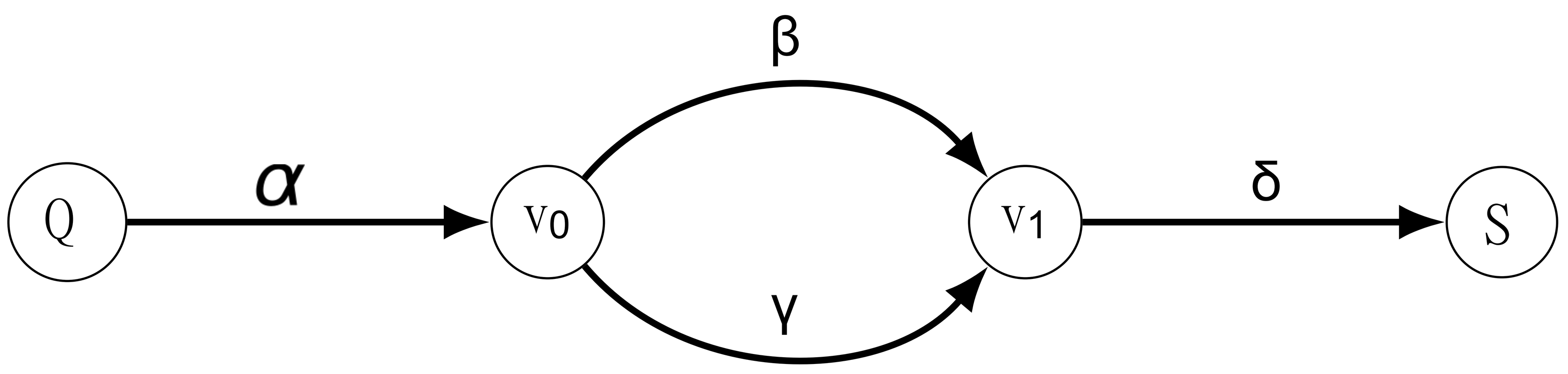
Nonlinear hyperbolic systems of balance laws in one space dimension model the flow inside the pipes or canals of the network. They are coupled through nonlinear coupling conditions at junctions that also model compressors, pumps or valves. The problem is a nonlinear boundary control problem for hyperbolic balance laws, e.g. for gas flow in a pipe *e* given by isothermal Euler equations

$$\partial_t \begin{pmatrix}\rho_e\\ \rho_e v_e\end{pmatrix} + \partial_x \begin{pmatrix}\rho_e v_e\\ \rho_e v_e^2 + p(\rho_e)\end{pmatrix} = \begin{pmatrix}0\\ -\lambda\,\rho^e\,|v^e|\,v^e\end{pmatrix},$$

where p is the pressure, $\rho > 0$ is the gas density, v is the gas velocity and λ is the friction coefficient.

Example for Node conditions

Conservation of mass and continuity of pressure. On figure: Circle with two border nodes



Convergence Results

With appropriate pointwise coupling conditions between the original system and the observer system it can be shown that the H^1-norm of the difference system with respect to the Riemann invariants decays exponentially on the graph from the figure. This allows in particular the treatment of networks with cycles, if nodal couplings are introduced in the vertices.

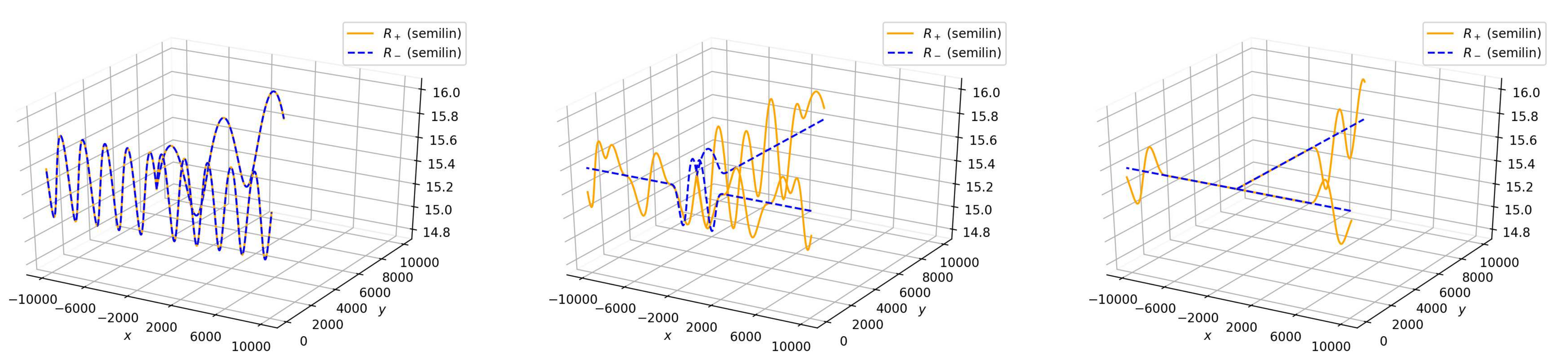
For a star shaped network we have shown that for a semilinear simplification of the isothermal Euler equations the state of the system with a source term can be stabilized in H^1 to a position of rest with feedback at the boundary nodes for arbitrary long pipes.

Numerical Results

The results for the star shaped network were supplemented by simulations for the semilinear and quasilinear case as shown in the figures.

Continuous initial data without friction

Snapshots of the semilinear solution at times t = 0s, t = 28s and t = 56s



Ongoing and Further Research

Data assimilation with Distributed Data in the Tubes

- What is the class of coupling conditions compatible with relative energy given pressure and velocity data for the entire domain?
- Can an observer be used in the case of partial density or velocity data?
- Can the theoretical understanding of data assimilation techniques be combined with numerical methods to better analyze measurement error behavior?
- How can we take into account the uncertainty of measurement data?

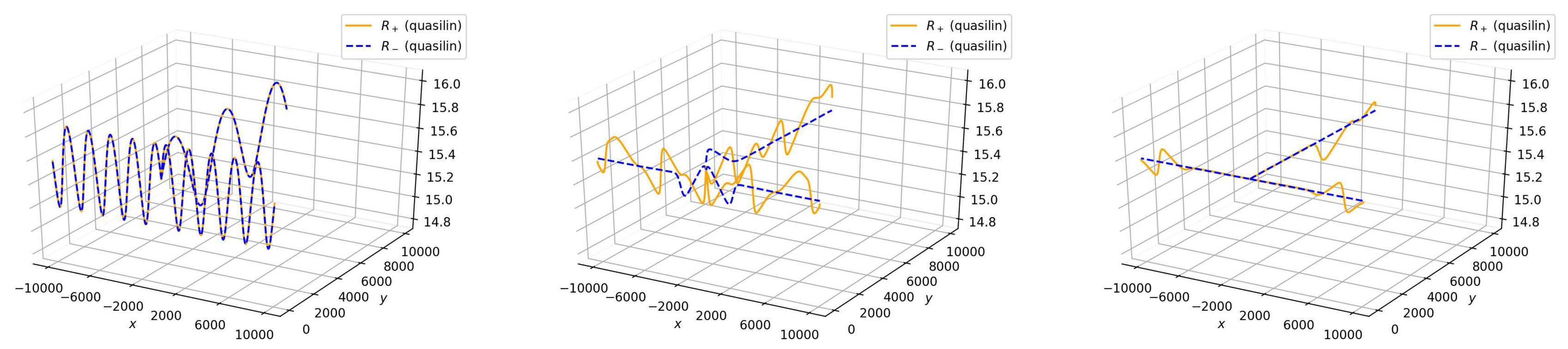
Data assimilation with Data only in Nodes

First, consider the system for a single pipe with boundary conditions at both ends x = 0 and x = L. A twin system of the same form is added, where one estimates the unknown initial state.

- What are Dirichlet boundary conditions for the difference system such that its norm decreases exponentially?
- What is the minimal time for exact synchronization of both systems?
- How can the stabilization results be generalized to networks and complemented by numerical implementations?

Continuous initial data without friction

Snapshot of the quasilinear solution at times t = 0s, t = 28s and t = 56s



Transition between Observation Types

- How is point observation related to distributed observation?
- Does a distributed observation with a support ω_ϵ contracting to a point for $\epsilon \to 0$ converges to a point control?

Results of previous Projects

Development of the Luenberger Observer

- Introduced by Luenberger in 1966
- Evolution of the observer under the name *Nudging*

Feedback stability in semi- and quasi-linear hyperbolic problems

- Results for the wave equation [1,2]
- Quasi-linear models of viscous, incompressible flows [3]

Relative energy as a distance measure for nonlinear systems of hyperbolic conservation equations

- A priori [4] and a posteriori [5, 6] error estimates
- Isothermal Euler equations can be reformulated as Hamiltonian systems [7]
- Preventing shocks with boundary control of quasi-linear systems [8]

Selected Publications

[1] F. M. Hante, M. Sigalotti, M. Tucsnak. **On conditions for asymptotic stability of dissipative infinite-dimensional systems with intermittent damping.** J. Differ. Equ. (2012)

[2] M. Gugat, G. Leugering, K. Wang. **Neumann boundary feedback stabilization for a nonlinear wave equation: a strict H2-Lyapunov function.** Math. Control Relat. Fields (2017)

[3] A. Farhat, M. S. Jolly, E. S. Titi. **Continuous data assimilation for the 2D Bénard convection through velocity measurements alone.** Phys. D (2015)

[4] E. Feireisl, M. Lukácová-Medvidová. **Convergence of a mixed finite element finite volume scheme for the isentropic Navier-Stokes system via dissipative measure valued solutions.** Found. Comput. Math. (2018)

[5] J. Giesselmann, C. Makridakis, T. Pryers. **A posteriori analysis of full discrete method of lines discontinuous Galerkin schemes for systems of conservation laws.** SIAM J. Numer. Anal. (2015)

[6] C. Rohde, F. Meyer, J. Giesselmann. **A posteriori error analysis for random scalar conservation laws using the stochastic galerkin method**, IMA J. Numer. Anal. (2019)

[7] J. Giesselmann, C. Lattanzio, A. E. Tzavaras. **Relative energy for the Korteweg theory and related Hamiltonian flows in gas dynamics.** Arch. Ration. Mech. Anal. (2016)

[8] M. Gugat, G. Leugering. **Global boundary controllability of the Saint-Venant system for sloped canals with friction.** Ann. Inst. H. Poincaré Anal. Non Linéaire (2009)

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