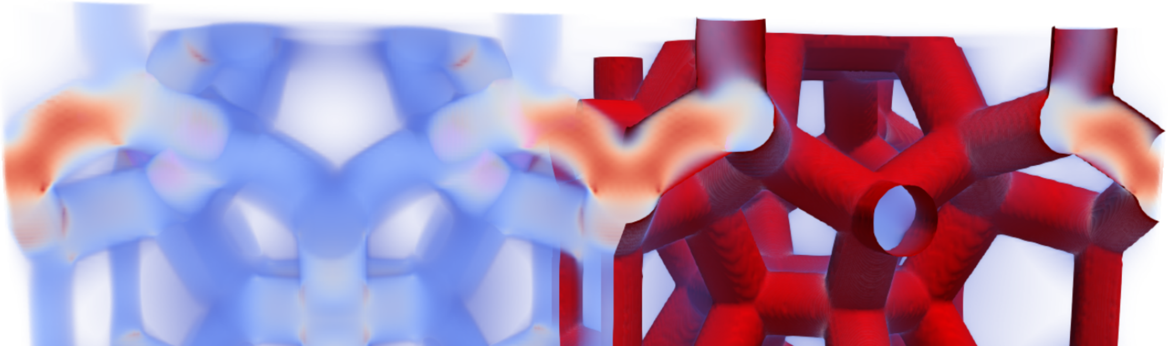
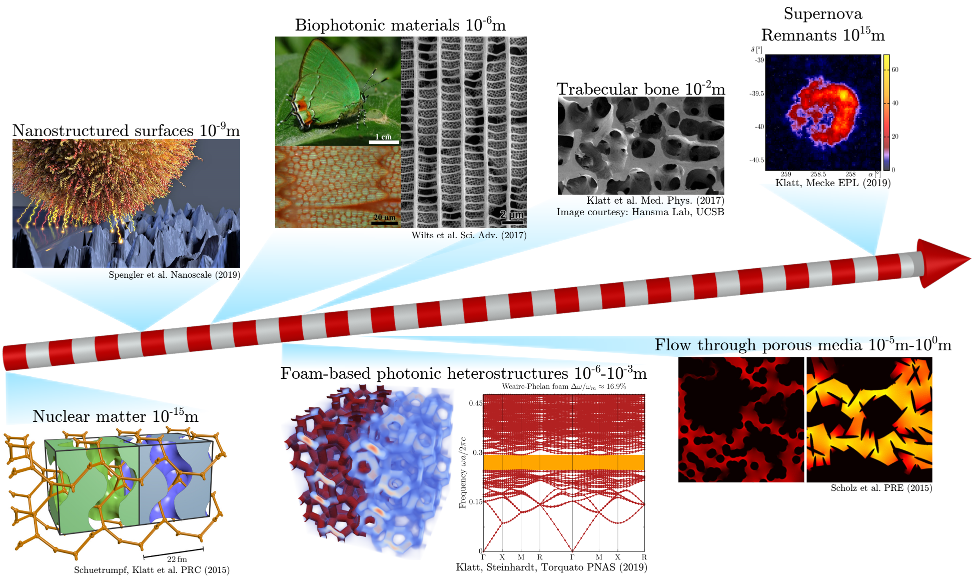


Photonic band gaps of optimized crystal and disordered networks

Michael Andreas Klatt

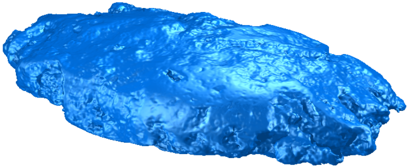
DCN Seminar, FAU, Erlangen, July 22 2021

Geometry and physics of complex spatial structures

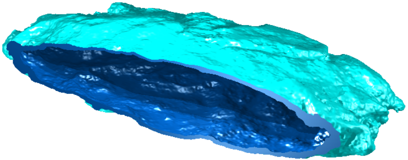


Minkowski functionals from integral geometry

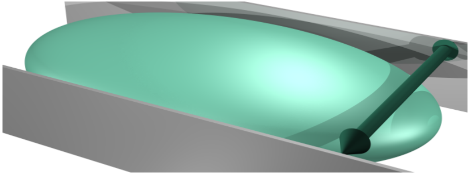
Volume



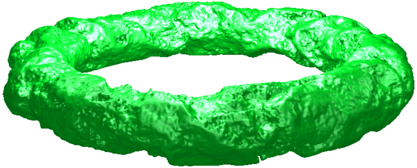
Surface area



Integrated mean curvature



Euler characteristic



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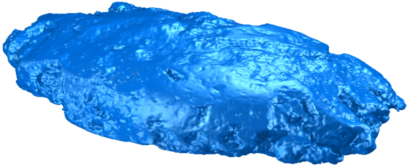

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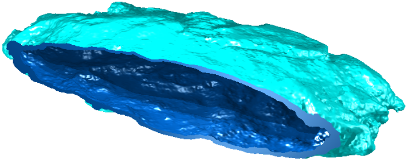

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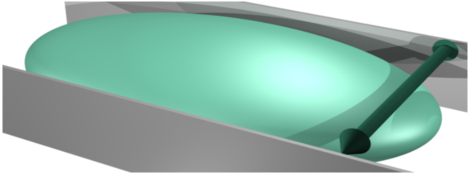

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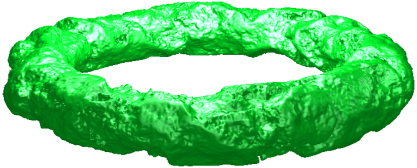

Additive functionals: $F(A \cup B) = F(A) + F(B) - F(A \cap B)$

Minkowski functionals from integral geometry

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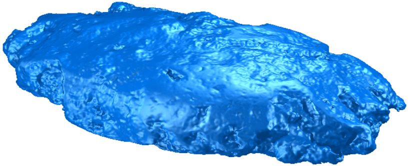

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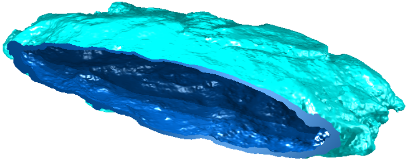

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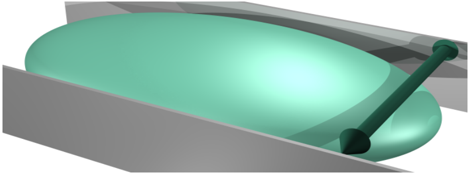

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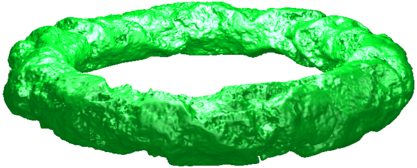
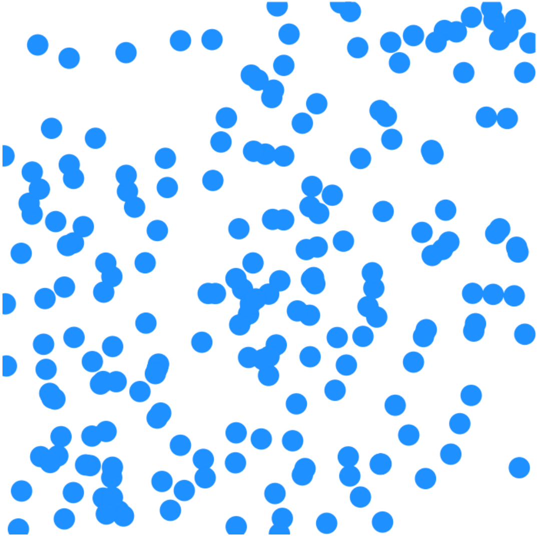


Additive functionals: $F(A \cup B) = F(A) + F(B) - F(A \cap B)$

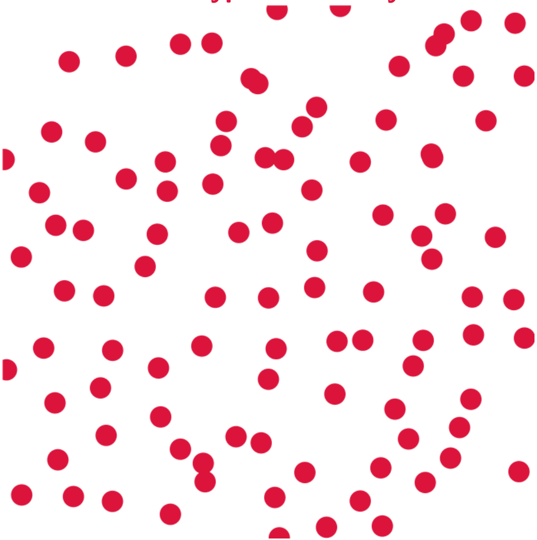
Hadwiger theorem (1957): Any additive, continuous, and motion invariant functional F on the set of convex bodies is a linear combination of Minkowski functionals.

Hidden long-range order

“Garden-variety” disorder



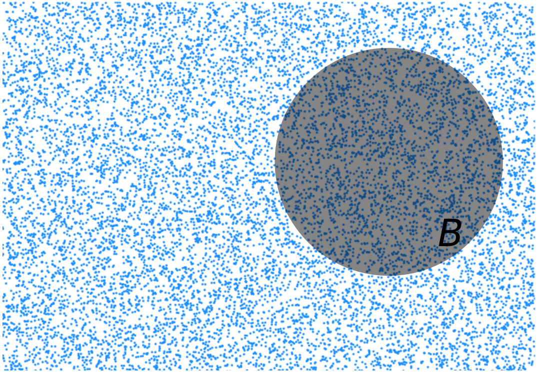
Hyperuniformity



Klatt, Last, Yogeshwaran. *Random Struct. Algor.* 57, 2020

Anomalous suppression of long-range density fluctuations

“Garden-variety” disorder



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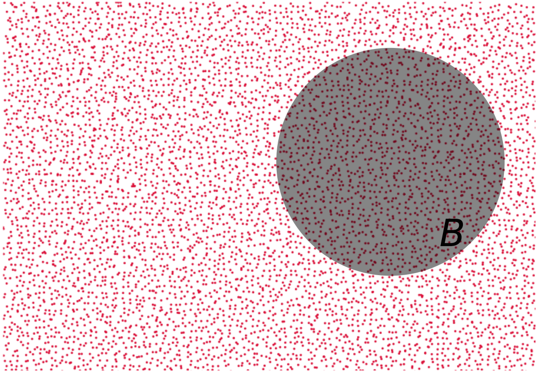

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$$\mathbb{V}\mathrm{ar}\left[\#\mathrm{Particles}\right] \sim \mathrm{Vol}[B]$$

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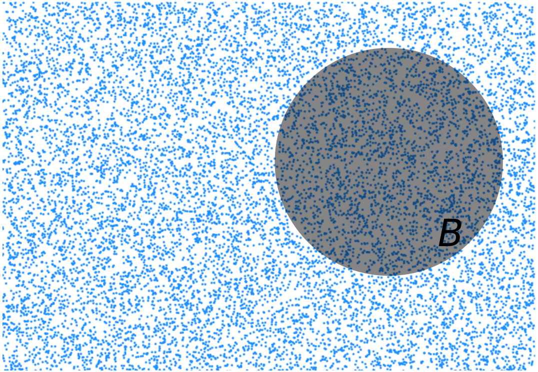

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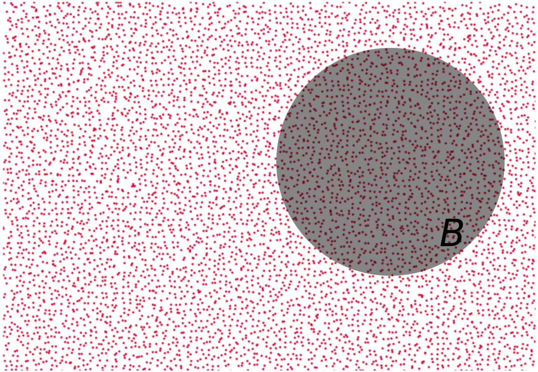

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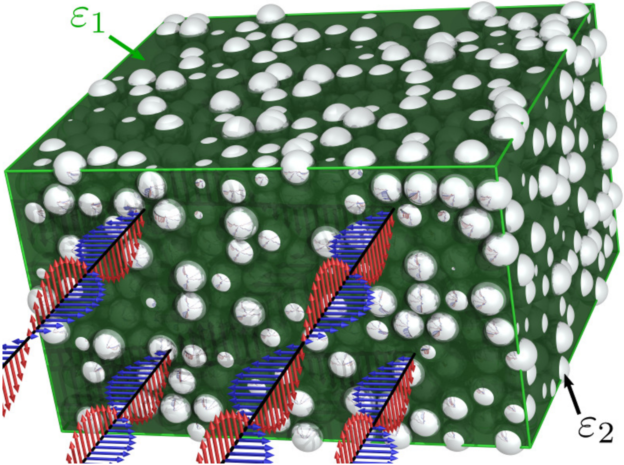
"Isotropic like liquid — homogeneous like crystal"

Torquato. *Phys. Rep.* 2018

Hyperuniform heterogeneous materials



Novel material design:

- Exponential decay of correlations
- Unique transport properties
- No dissipation of waves

Klatt, Torquato. *Phys. Rev. E* 2018

Torquato. *Phys. Rep.* 2018;

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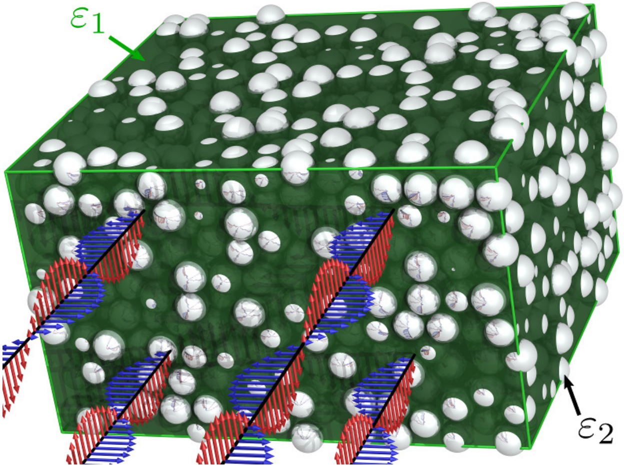

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- Photoreceptor cells in eyes of chicken
- Random self-organization
- Active matter

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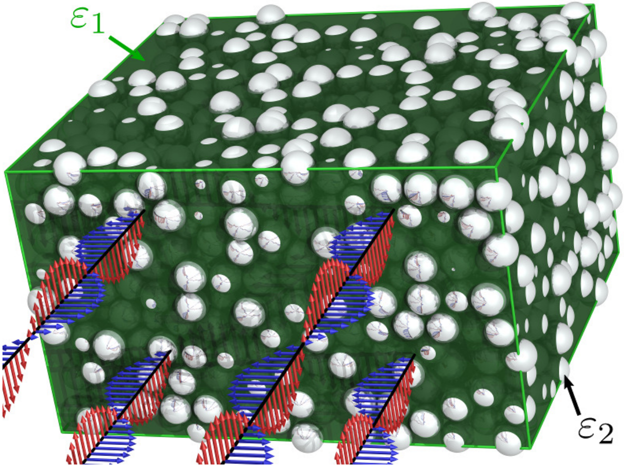

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Empirical Conjecture:
Hyperuniformity is a necessary condition for photonic band gaps

Klatt, Torquato. *Phys. Rev. E* 2018

Torquato. *Phys. Rep.* 2018;

Photonic band gaps of optimized networks

1. What are photonic band gaps?

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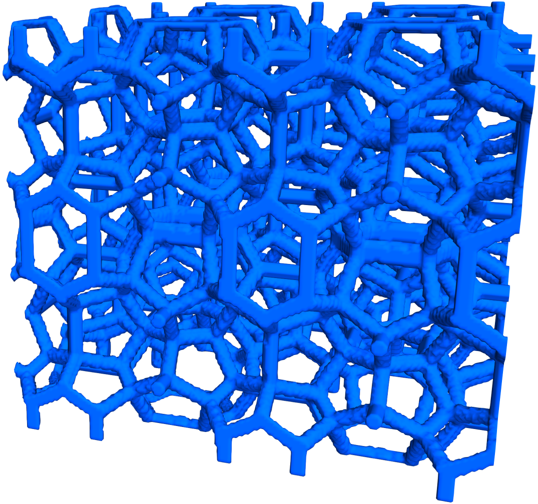
Photonic crystals: “Semiconductors of light”

Propagation of light prohibited for a range of frequencies in all directions

Yablonovitch. *PRL* 58, 1987; John. *PRL* 58, 1987



Klatt, Steinhardt, Torquato. *PNAS* 116, 2019

Computing photonic band gaps

Propagation of electromagnetic waves, including light, is goverend by Maxwell's equations:

$$\nabla \cdot \mathbf{H}(\mathbf{r},t) = 0 \qquad\qquad \nabla \times \mathbf{E}(\mathbf{r},t) + \mu_0 \frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} = 0$$

$$\nabla \cdot [\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t)] = 0 \qquad\qquad \nabla \times \mathbf{H}(\mathbf{r},t) - \varepsilon_0 \varepsilon(\mathbf{r}) \frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} = 0$$

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with

$\mathbf{r}$: position vector

t : time

$\mathbf{H}(\mathbf{r}, t)$: magnetic field

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ε_0 : constant vacuum permittivity

μ_0 : constant vacuum permeability

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where we here assume:

- no sources
- linear constitutive relations
- no material dispersion
- scalar dielectric function

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Because Maxwell's equations are linear, we can restrict our analysis to harmonic modes

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) e^{-i\omega t} \qquad \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) e^{-i\omega t}$$

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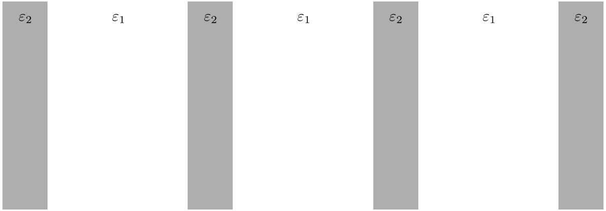
Thus, we obtain the master equation:

$$\nabla \times \left[\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right] = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r})$$

where $c := 1/\sqrt{\varepsilon_0 \mu_0}$ is the vacuum speed of light.

Layered medium: periodic in one direction



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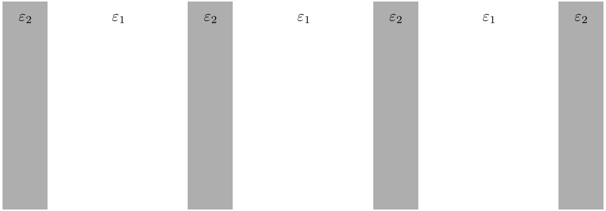

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Floquet mode:

$$\mathbf{H}(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u_{\mathbf{k}}(\mathbf{r})$$

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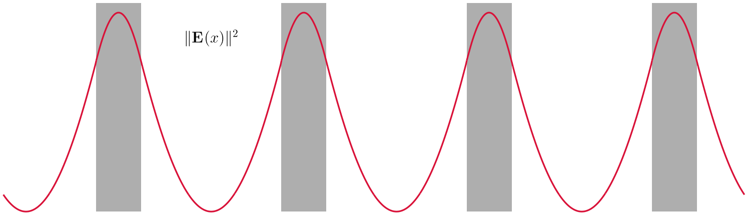

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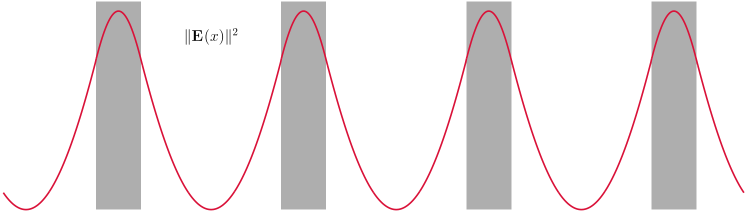

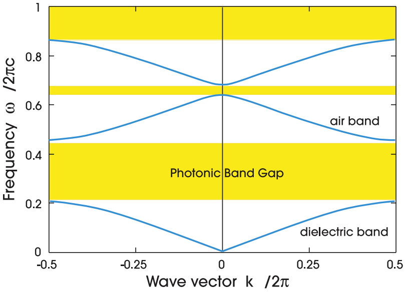

Source: Joannopoulos et al. 2008

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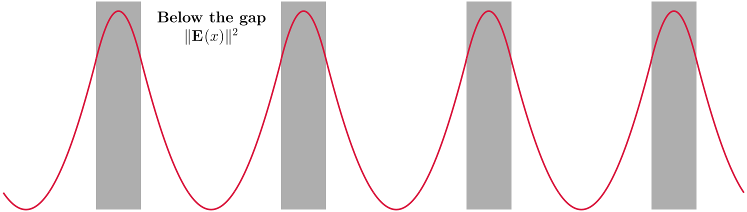

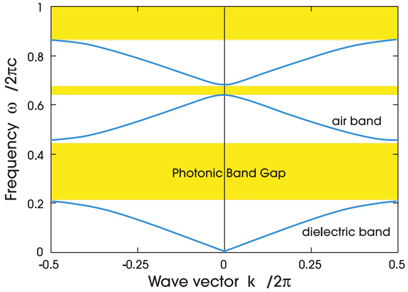

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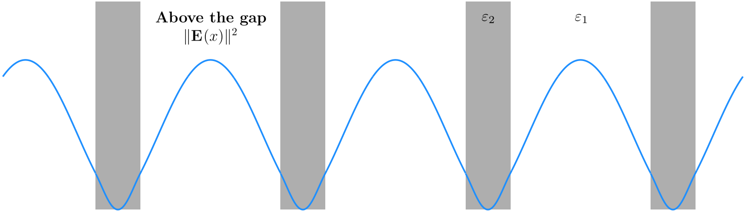

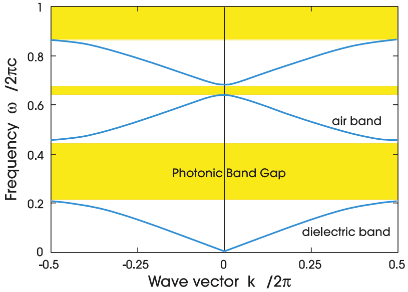

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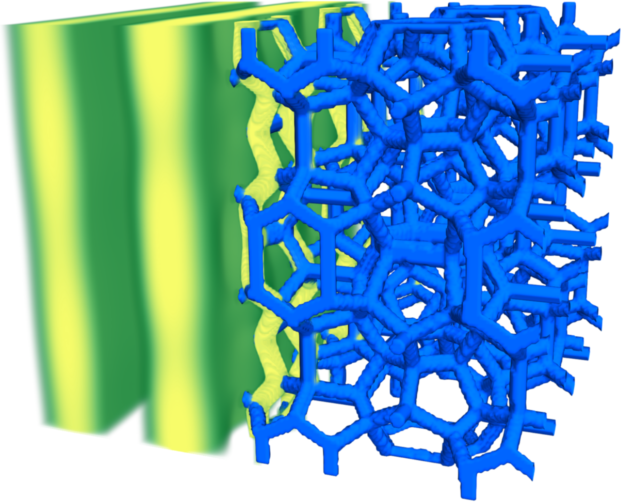
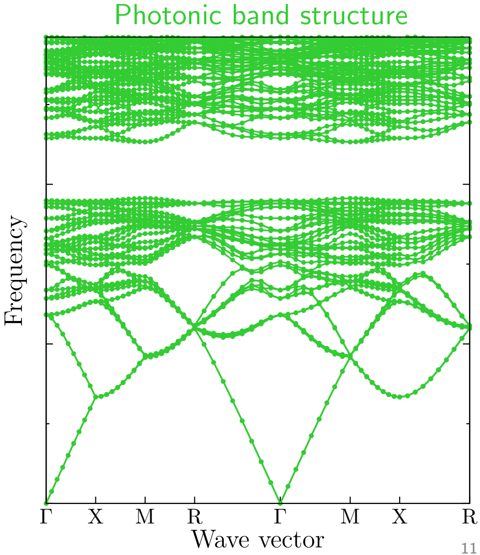
Photonic crystals: 3D network-like structures

Propagation of light prohibited for a range of frequencies in all directions



Klatt, Steinhardt, Torquato. *PNAS* 116, 2019



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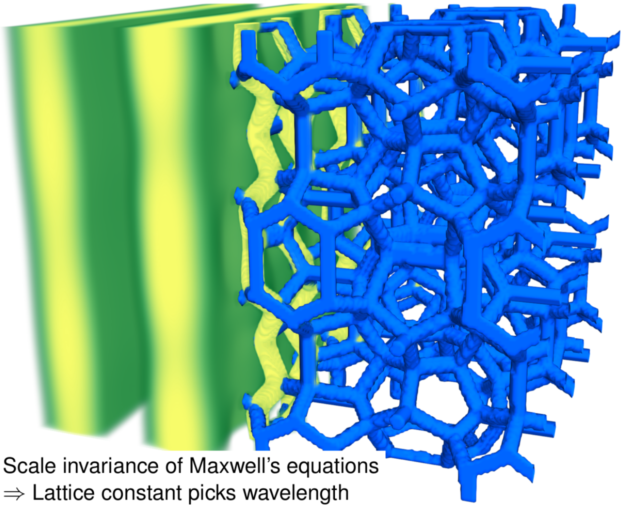
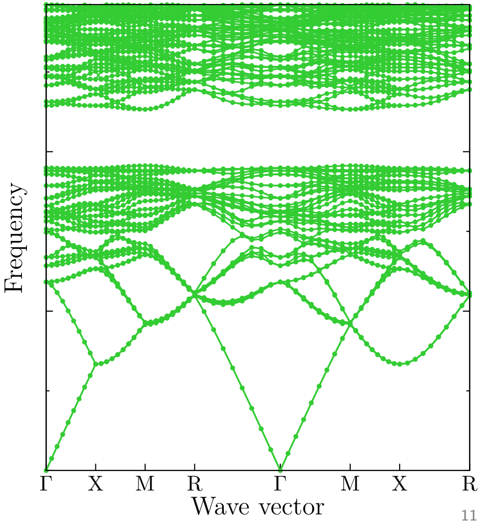
Propagation of light/infrared rad./THz prohibited for a range of frequencies in all directions



Scale invariance of Maxwell's equations
⇒ Lattice constant picks wavelength

Klatt, Steinhardt, Torquato. *PNAS* 116, 2019

Photonic band structure



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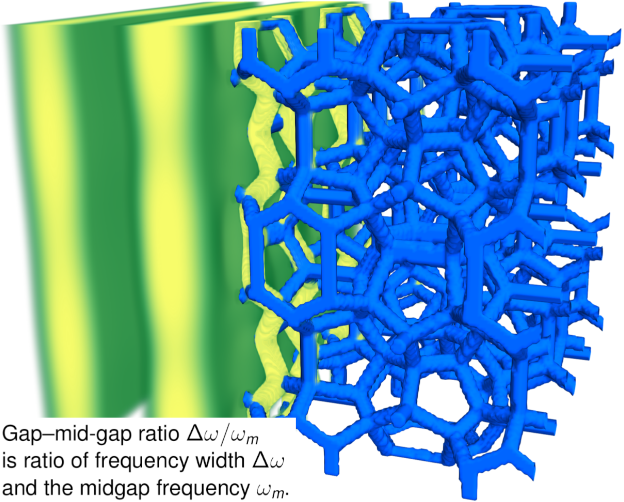
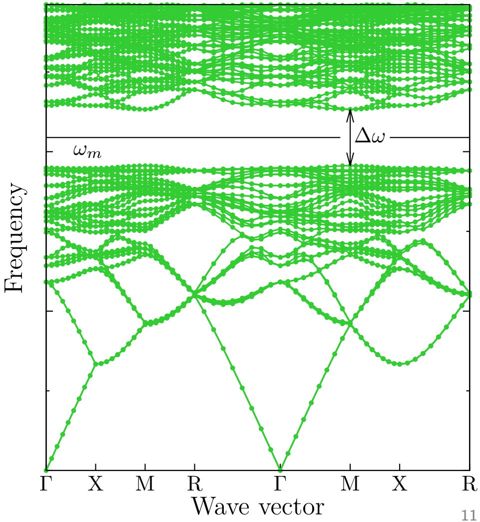


Gap–mid-gap ratio $\Delta\omega/\omega_m$ is ratio of frequency width $\Delta\omega$ and the midgap frequency ω_m.

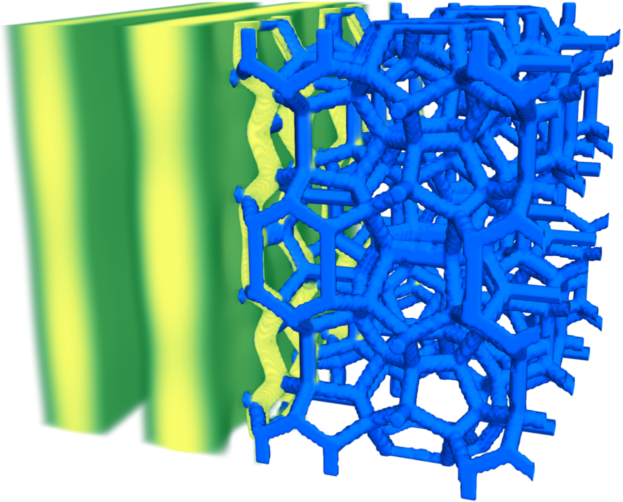
Klatt, Steinhardt, Torquato. *PNAS* 116, 2019

Photonic band structure



“Photonics + Foam = Phoamtonics”

First complete photonic band gap of foam-based heterostructure



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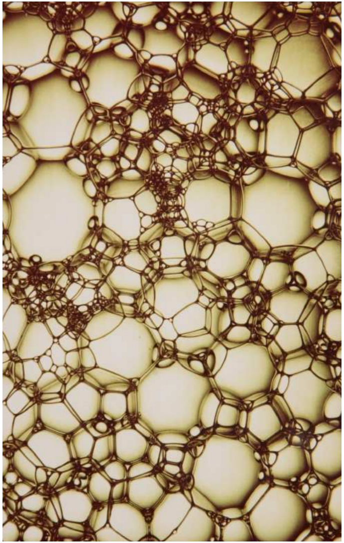
4. Open problem: band folding

What is a foam?



Source: Michael Boran
from Weaire, Cox, Brakke, 2005

Physics:

- Dispersion of a gaseous phase in a liquid or solid phase
- Liquid fraction:
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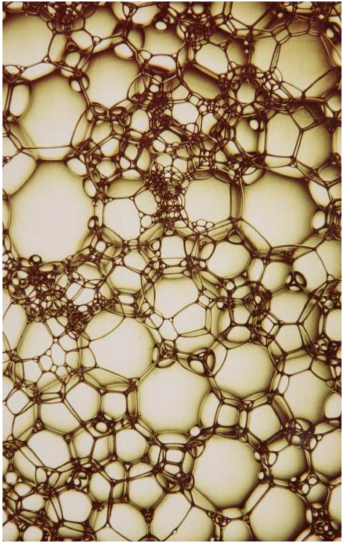
What is a foam?



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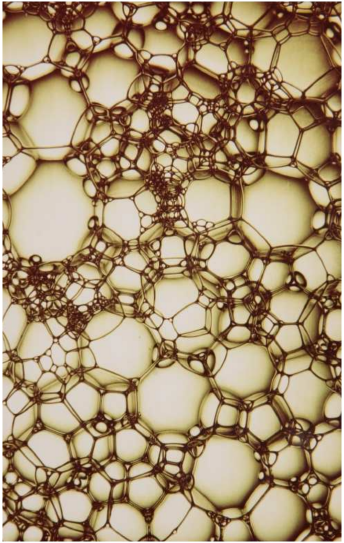

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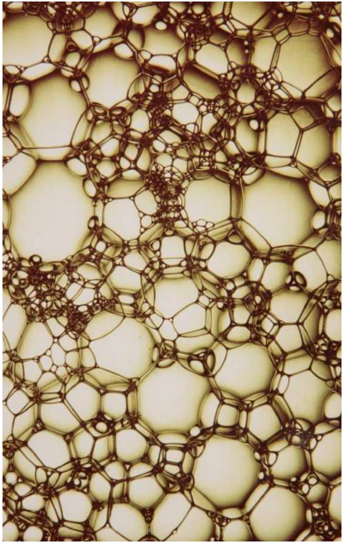
- Dry foam: vanishing liquid fraction

Mathematics:

- Dry foams locally minimize the surface area of their cells subject to volume constraints.

What is a foam?



Source: Michael Boran
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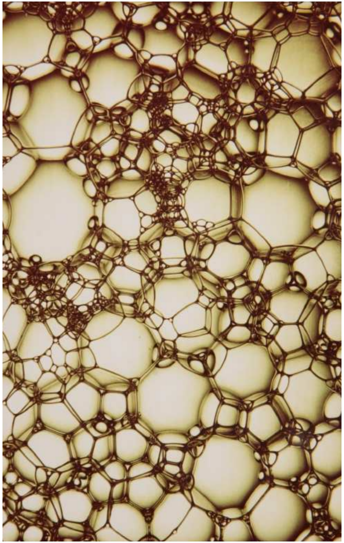

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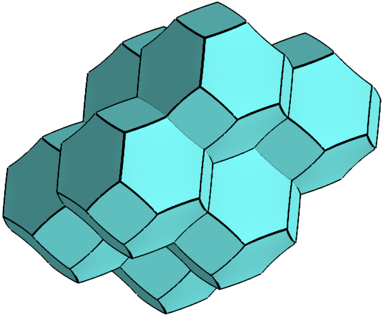

Conjecture: Relaxation of Voronoi cells of body-centered cubic (bcc) lattice

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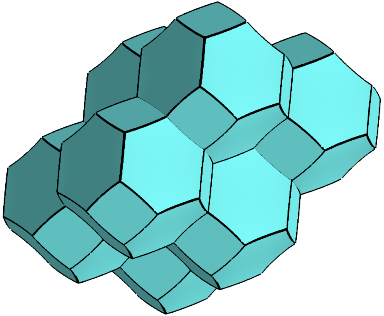

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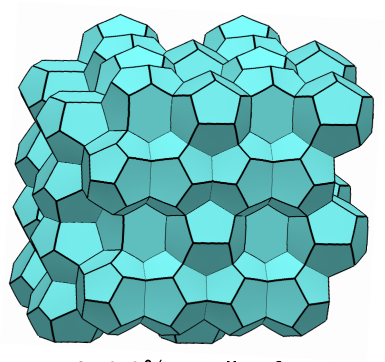

Surface area is 0.3% smaller for Weaire-Phelan than Kelvin foam

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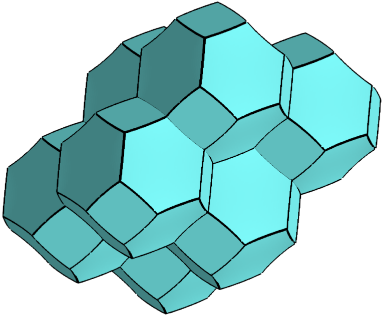

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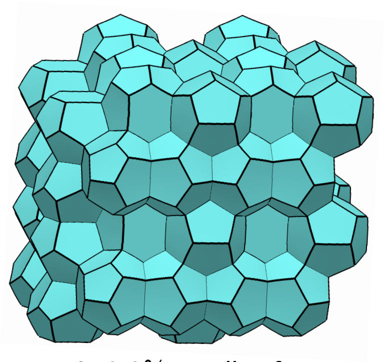

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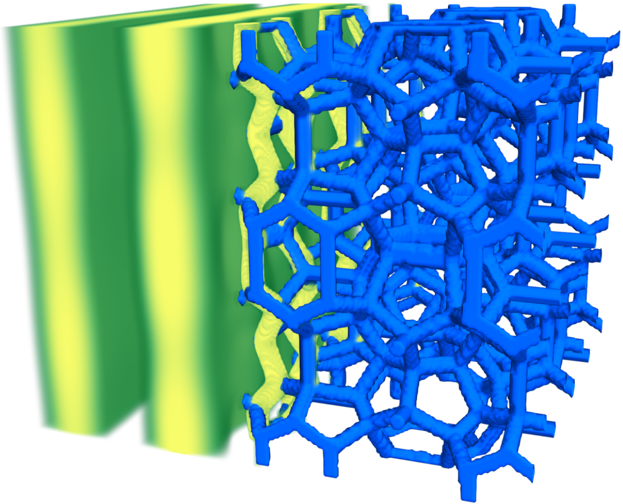
“Photonics + Foam = Phoamtonics”

Complete photonic band gap of foam-based heterostructure



Edges of Weaire-Phelan foam

Klatt, Steinhardt, Torquato. *PNAS* 116, 2019

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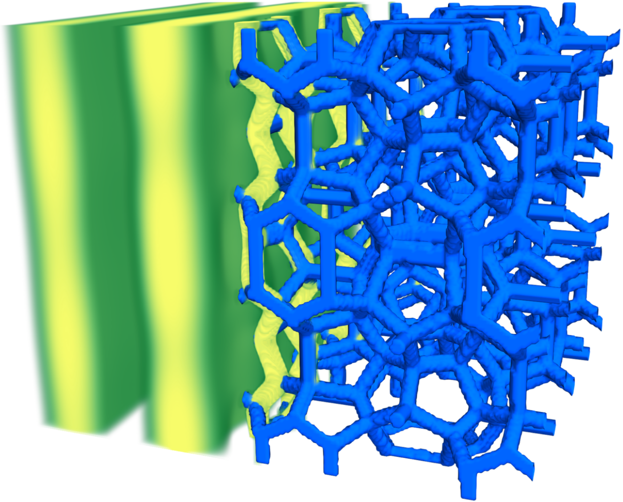

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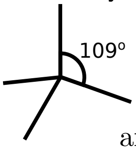
Plateau’s laws for dry foam
(with vanishing liquid fraction)
guarantee exclusively tetrahedral vertices



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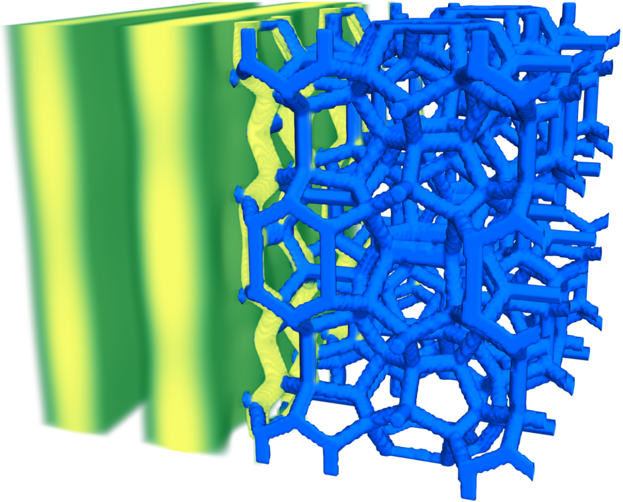

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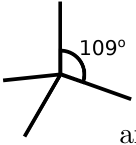

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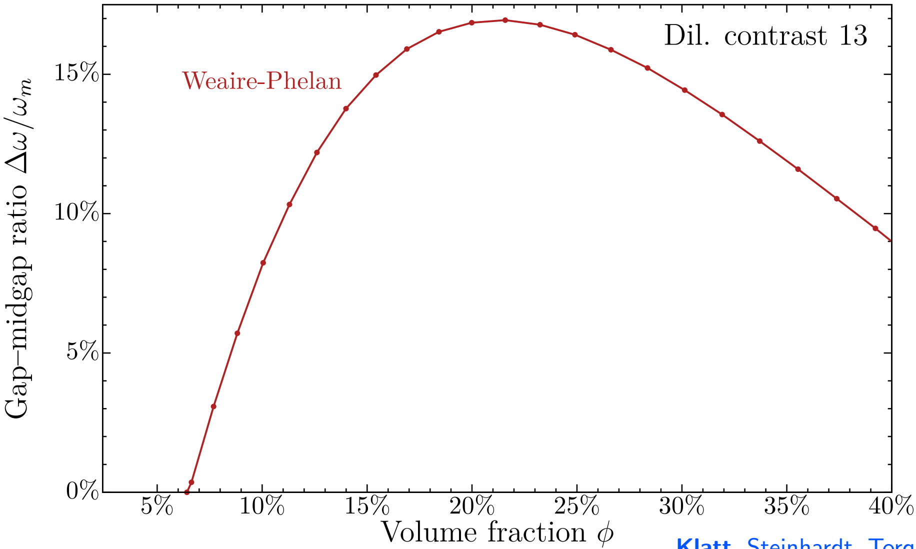
Tetrahedral vertices empirically known to be advantageous for photonic band gaps

Klatt, Steinhardt, Torquato. *PNAS* 116, 2019

Gap size varies with volume fraction and dielectric contrast



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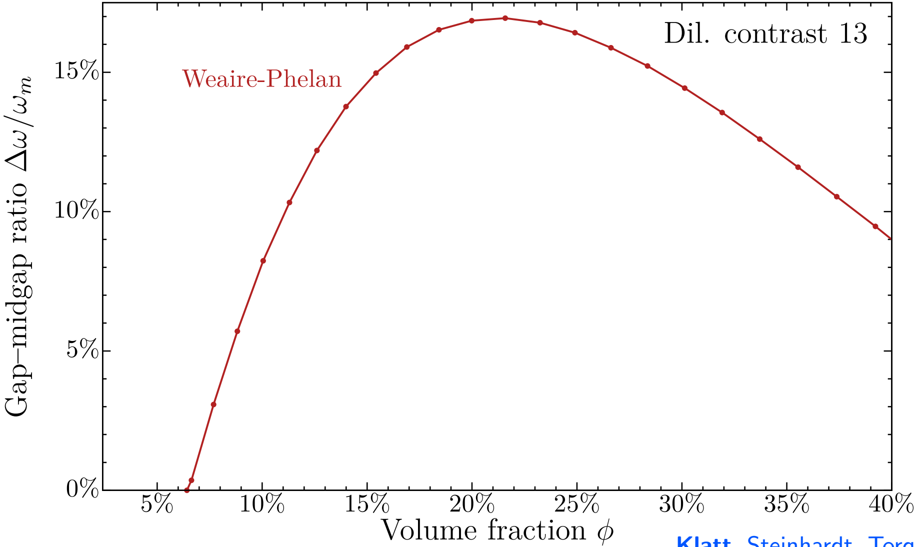

Open cells

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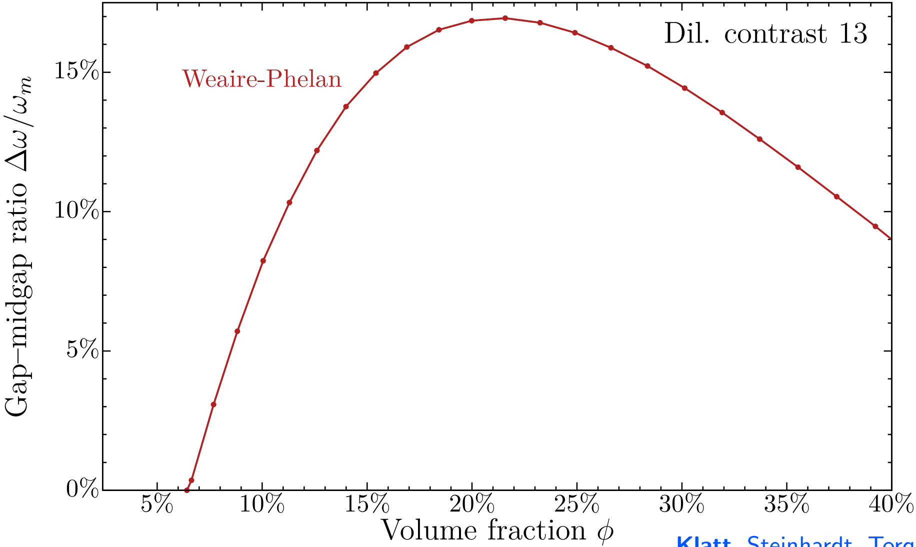

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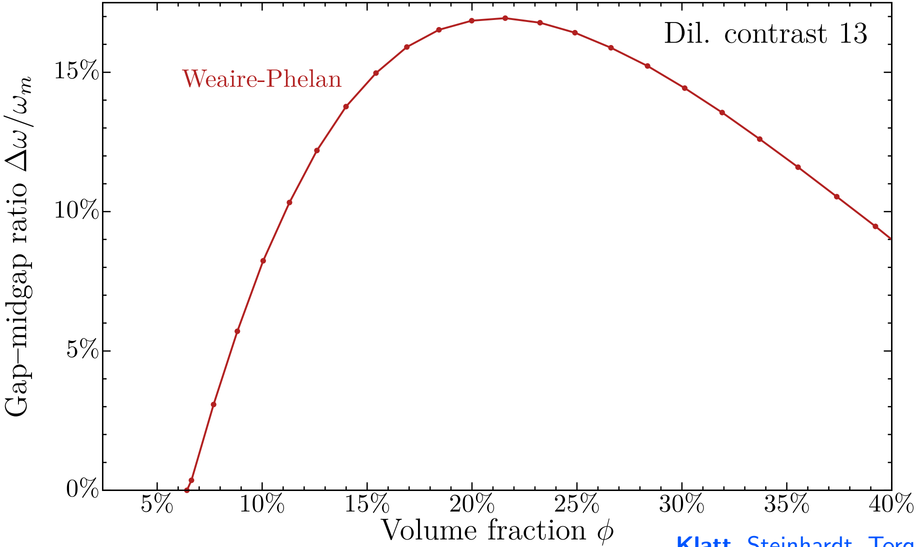

Open cells

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Coat edges

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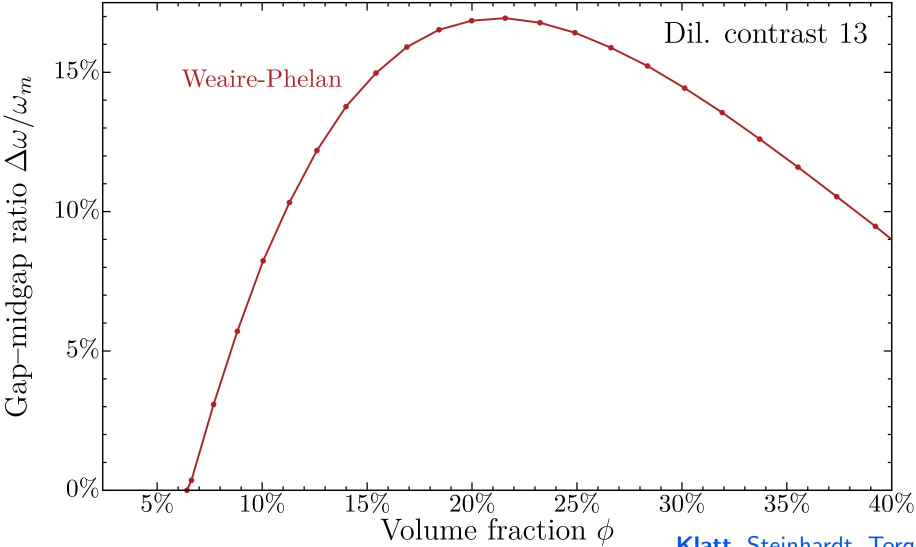

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Muller et al.

Adv. Opt. Mater. 2014

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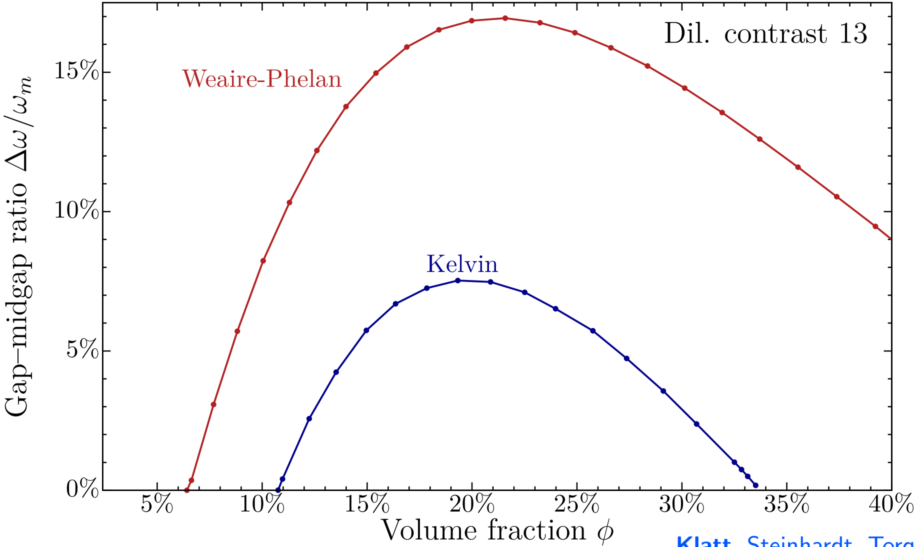

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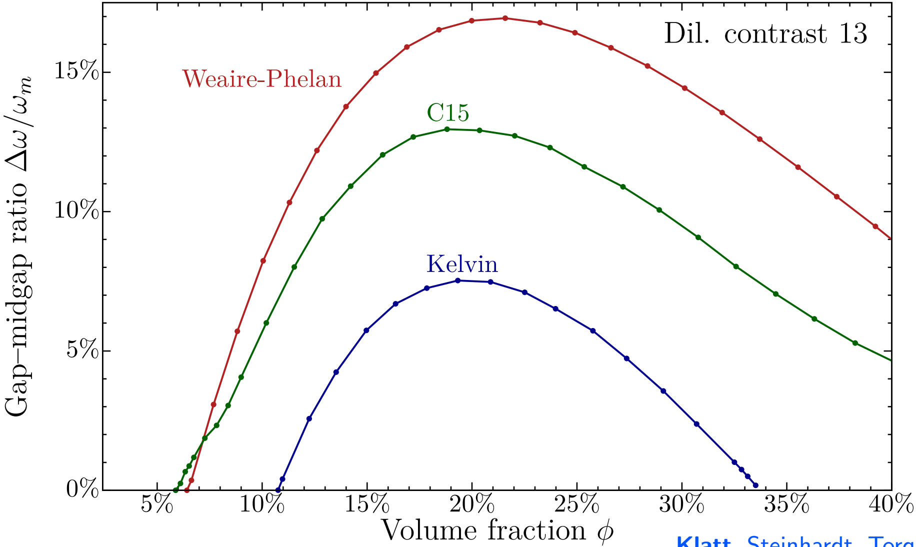

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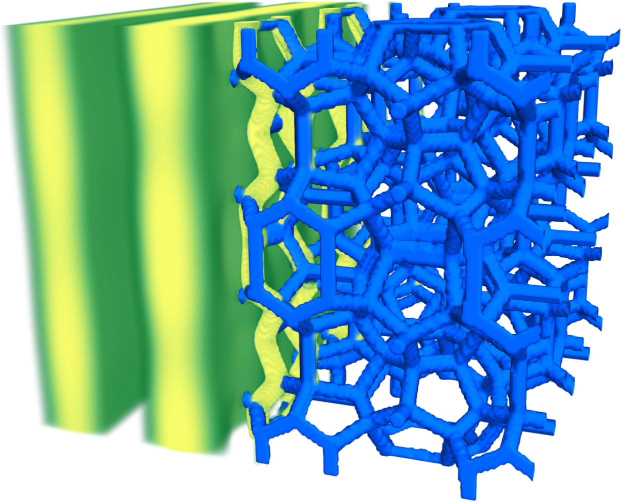

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Utility of foams for applications

- Multifunctionality
- Self-organization
- High degree of isotropy among crystal structures

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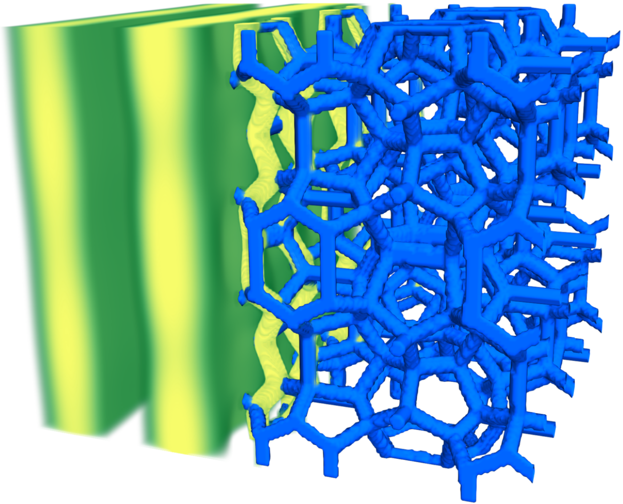

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Applications at visible wavelengths are challenging with current technology

Standard techniques of solid open-cell foams for cell sizes in sub-millimeter regime $\Rightarrow$ THz radiation

Photonic band gaps of optimized networks

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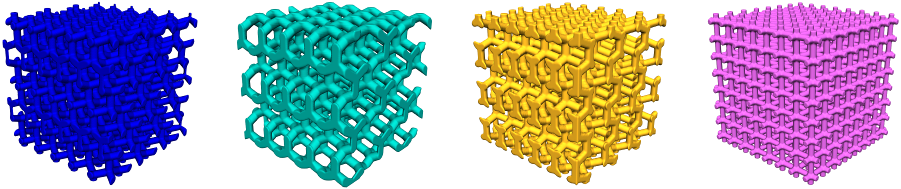
4. Open problem: band folding

Networks with a variety of topologies and symmetries



Crystal networks

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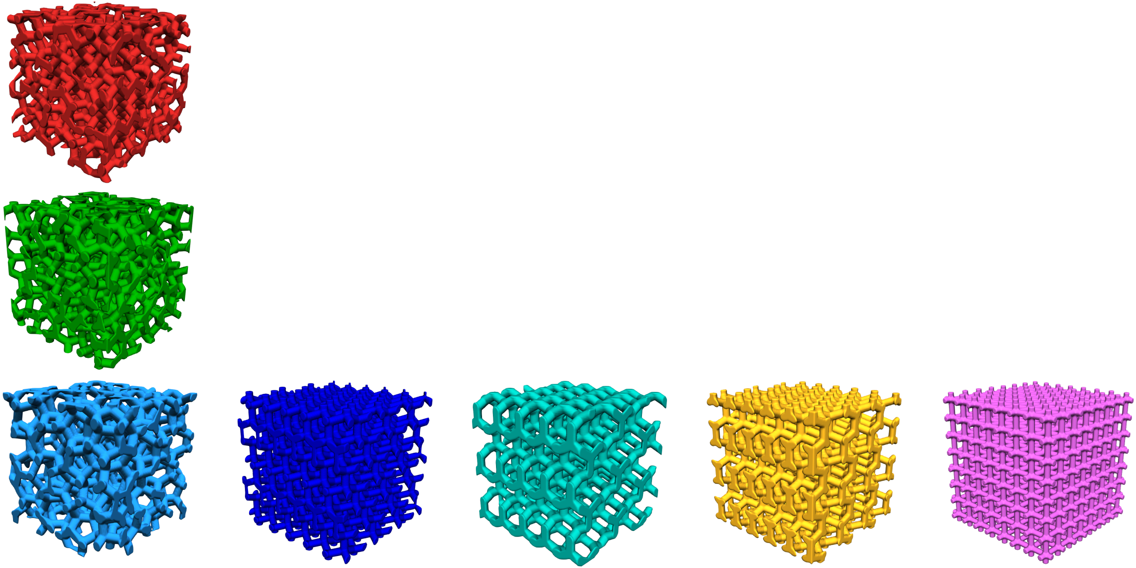

Disordered networks

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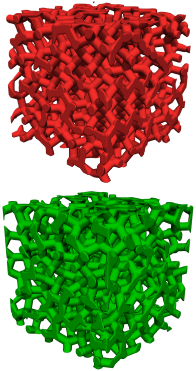
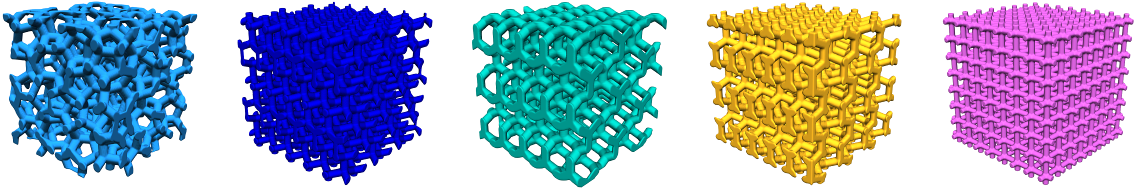


Networks of rods

with radius R and
dielectric contrast ε

$$\Delta(\varepsilon) := \max_{R \geq 0} \left\{ \frac{\Delta\omega}{\omega_m}(R, \varepsilon) \right\}$$



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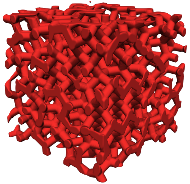

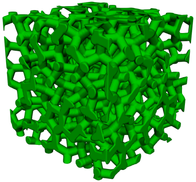

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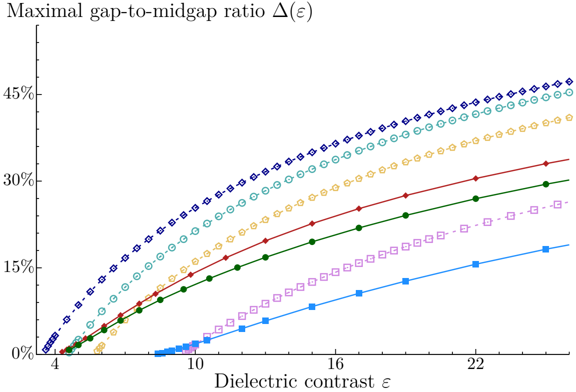

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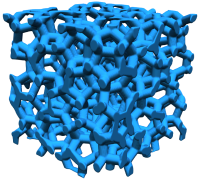

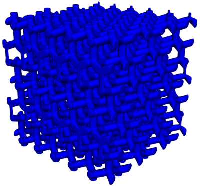

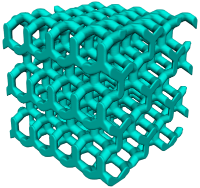

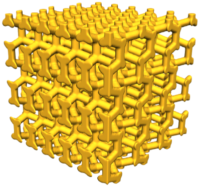

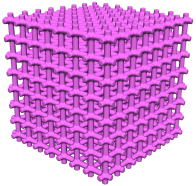

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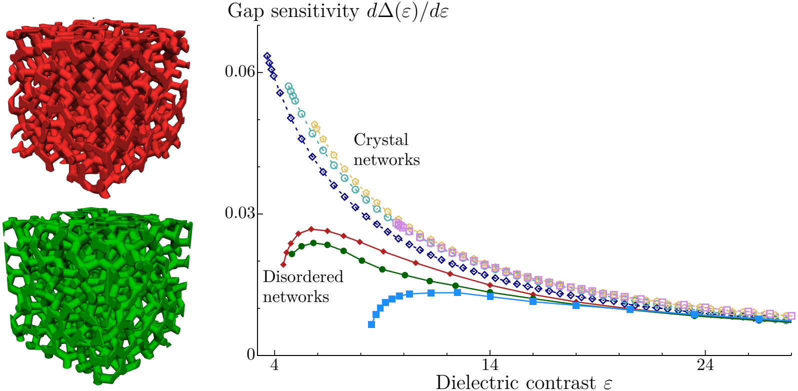

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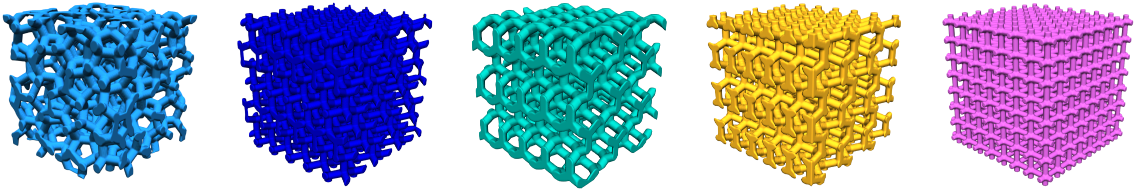

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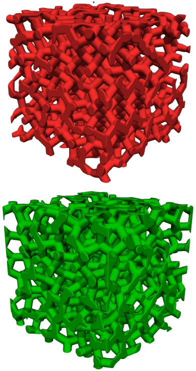

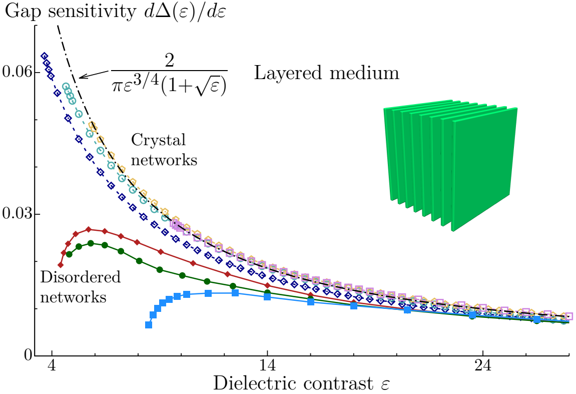

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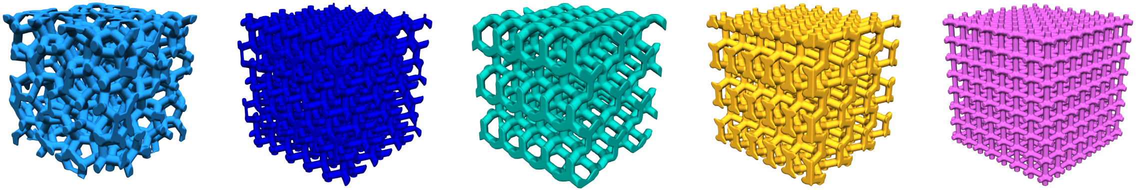

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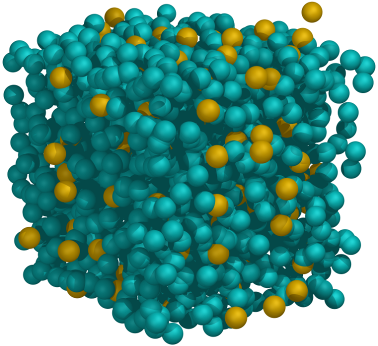
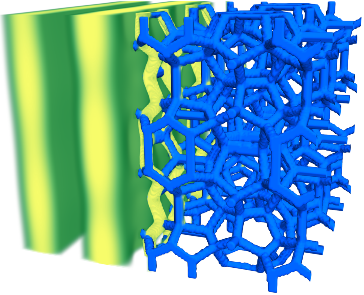
Conclusion

Hyperuniformity



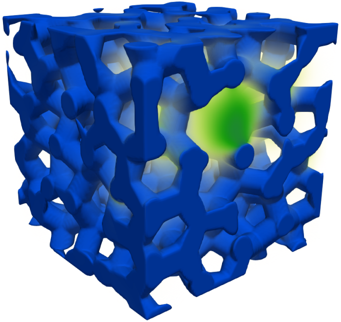
Isotropic like liquids, homogeneous like crystals

Phoamtonics



Turning foams into photonic networks to “mold the flow of light”

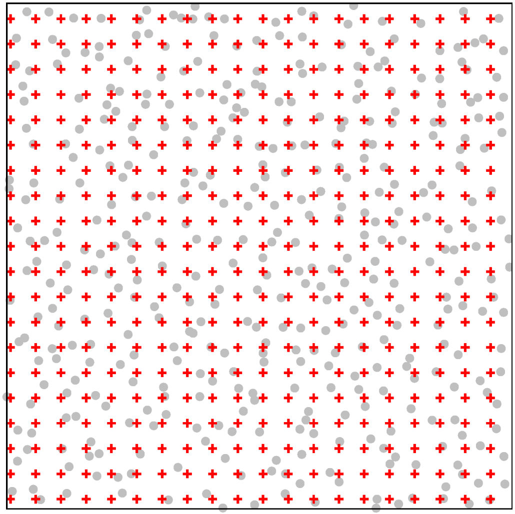
Universal gap sensitivity



For optimized gap size of crystal and disordered networks

Back up

Stable matching of point patterns



- Start with hyperuniform lattice $\mathbb{Z}^d$ and
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  for example, complete spatial randomness

Klatt, Last, Yogeshwaran. *Random Struct. Algor.* 57, 2020

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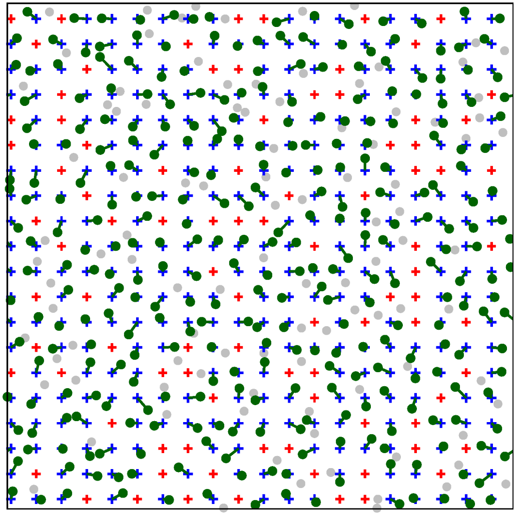

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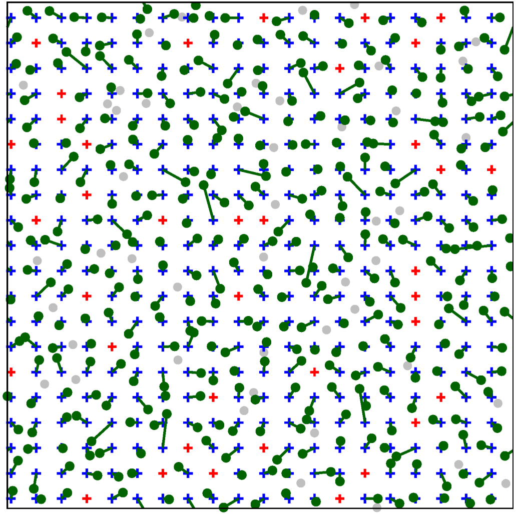

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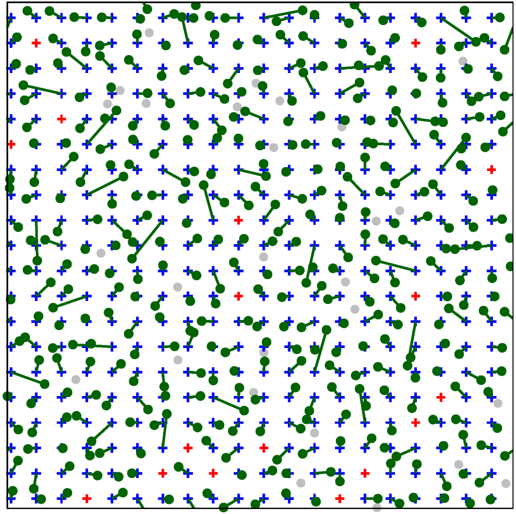

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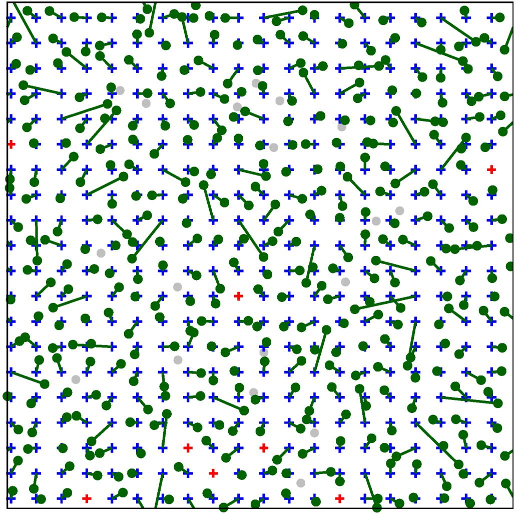

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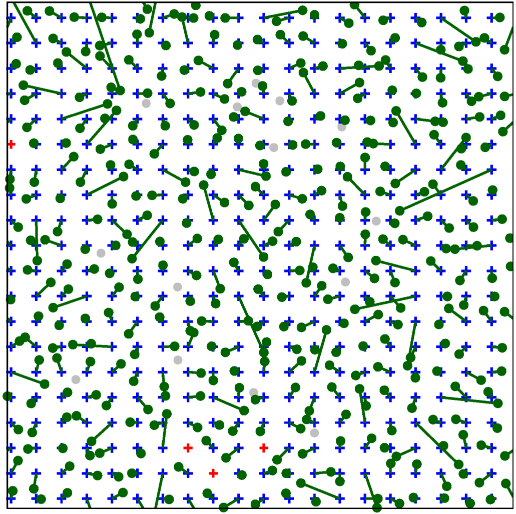

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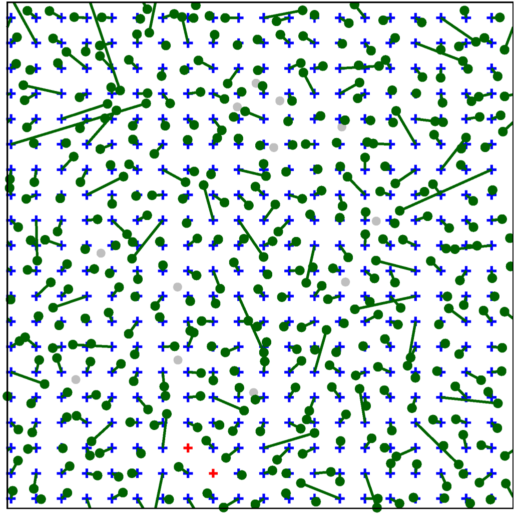

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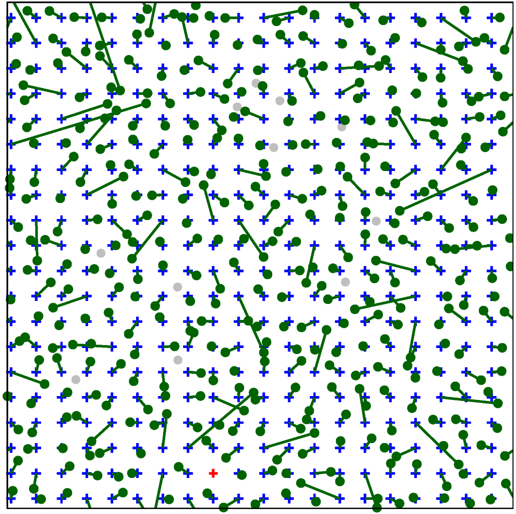

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Klatt, Last, Yogeshwaran. *Random Struct. Algor.* 57, 2020

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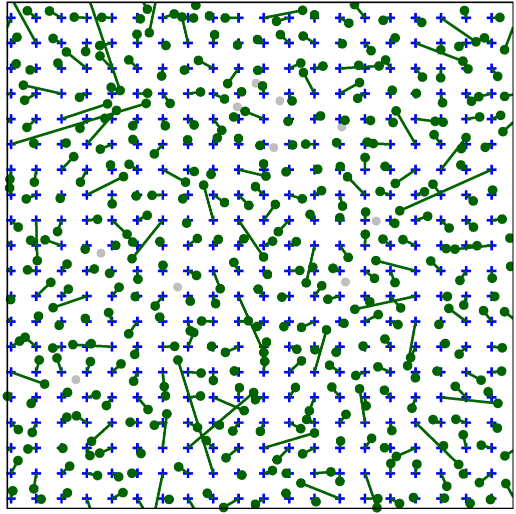

- Start with hyperuniform lattice $\mathbb{Z}^d$ and non-hyperuniform point process with, on average, more than one point per unit area, for example, complete spatial randomness
- Iteratively match points that are mutual nearest neighbors
- Unique stable matching (in sense of Gale and Shapely, 1962)

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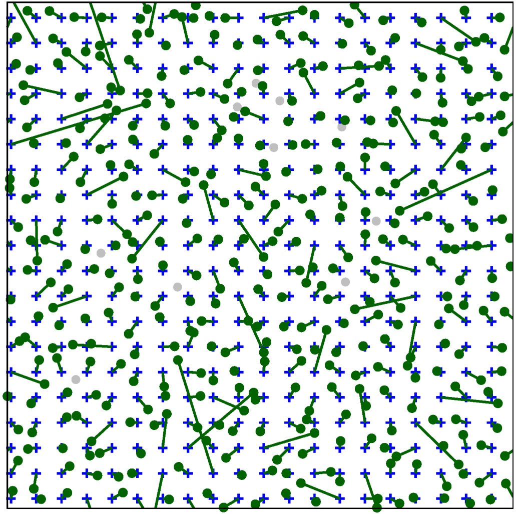

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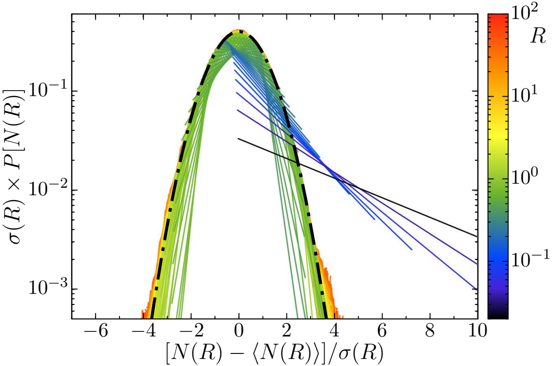
Theorem [Klatt, Last, Yogeshwaran 2020]
New process of matched (green) points is hyperuniform

Klatt, Last, Yogeshwaran. *Random Struct. Algor.* 57, 2020

Systematic characterization of density fluctuations

Number distribution: higher-order moments



Inferred presence of higher-body correlations

Torquato, Kim, **Klatt**. *Phys. Rev. X* 11:021028, 2021

Systematic characterization of density fluctuations

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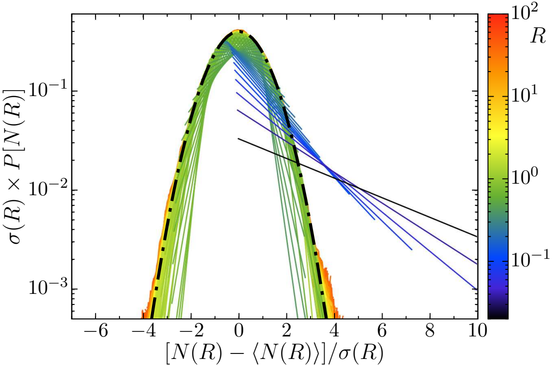
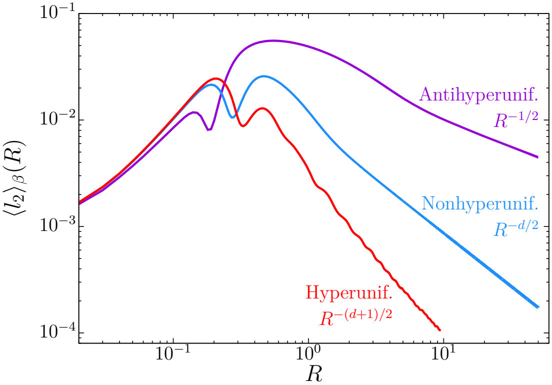


Inferred presence of higher-body correlations

Gaussian distance metric



Different scalings for hyperuniform and non-hyperuniform models

Torquato, Kim, **Klatt**. *Phys. Rev. X* 11:021028, 2021