

Control and Design: Model and Data Driven

2021⁺
**Summer Short Course-Control,
Machine Learning and Numerics**

Third-year Mathematical Science in Frankfurt (HSN) will hold the summer short course between July 4 - 28, 2021. It is organized by Prof. Dr. Enrique Zuazua, Friedrich-Alexander University, Erlangen-Nürnberg, Germany.

Course: Control, Machine Learning and Numerics
Date: July 4 - 28, 2021
Instructors: Prof. Dr. Enrique Zuazua, Friedrich-Alexander University, Germany
Participants: 100 students, 100 researchers, 100 teachers, 100 staff

Schedule:

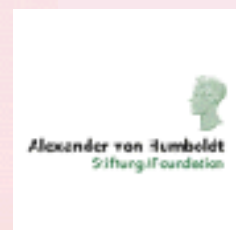
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				July 9 10:00 - 12:00	July 10 10:00 - 12:00
		July 11 10:00 - 12:00		July 12 10:00 - 12:00	July 13 10:00 - 12:00
July 14 10:00 - 12:00	July 15 10:00 - 12:00	July 16 10:00 - 12:00	July 17 10:00 - 12:00	July 18 10:00 - 12:00	July 19 10:00 - 12:00
July 20 10:00 - 12:00					


Chair of Dynamics, Control and Numerics
Alexander von Humboldt Professorship
Expert in Applied Mathematics: Partial
Differential Equations, Systems
Control, Numerical Analysis and
Machine Learning

FAU University of Applied Sciences
Ernst-Reuter-Str. 15
91054 Erlangen, Germany
Tel: +49 (0) 91 31 85-1234
Fax: +49 (0) 91 31 85-1234

Enrique Zuazua


Chair of Dynamics, Control and Numerics
Department of Data Science
FAU
Erlangen, Germany
<https://caa-avh.nat.fau.eu>






Chair
**DYNAMICS, CONTROL
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 Chair for Dynamics, Control and Numerics
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 FAU Erlangen - Nürnberg (Germany)
DDS, Department of Data Sciences

 Alexander von Humboldt
Stiftung/Foundation

 Prof. Dr. Enrique Zuazua
Alexander von Humboldt Professor at FAU

The World as an accumulation of 'shapes in motion'



We are passionate people working actively in the broad area of Applied Mathematics, developing and applying methods of Mathematical and Computational Mathematics to model, understand, design and control the dynamics of various phenomena arising in the interface of Mathematics with Engineering, Physics, Biology and Social Sciences:



Biology and Social Sciences
Control of diffusion models arising
in Biology and Social Sciences



Traffic & Energy
Hyperbolic models arising in traffic
flow and energy transport



Material Sciences
Optimal design in Material
Sciences



Design & Control
The interplay between discrete and
continuous modeling in design and
control



Identification
Inversion and parameter
identification



PDE
Analysis of Partial Differential
Equations (PDE) and Fractional PDE



Multi-agent Systems
Modeling and control of multi-
agent systems



Micro-Macro
Micro-macro limit processes



Turnpike
The emergence of turnpike
phenomena in long-time horizons



Tools
Development of new computation
tools and software

Taking Maths & Research beyond!

<https://caa-avh.nat.fau.edu/>



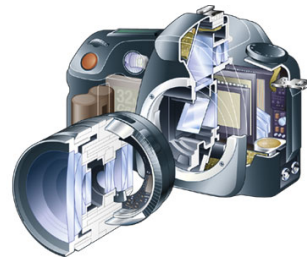
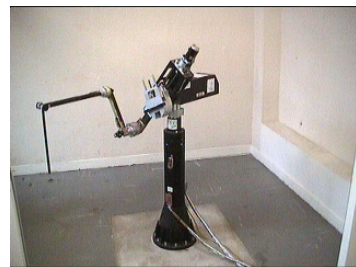
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- 6 Perspectives

Control theory and applications

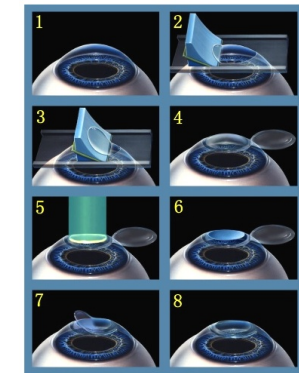
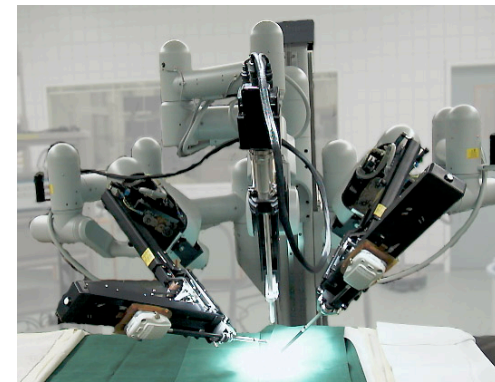
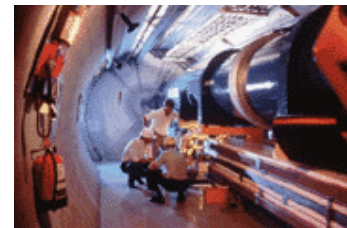
Mechanics

Vehicles (guidance, dampers, ABS, ESP, ...),
Aeronautics, aerospace (shuttle, satellites), robotics



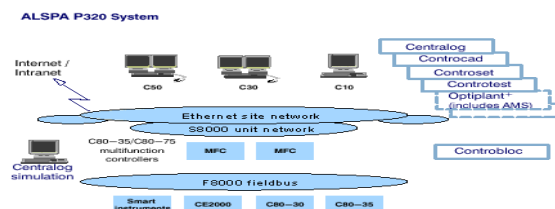
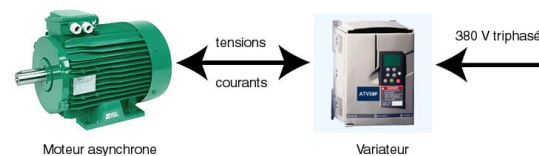
Biology, medicine

Predator-prey systems, bioreactors, epidemiology,
medicine (peacemakers, laser surgery)



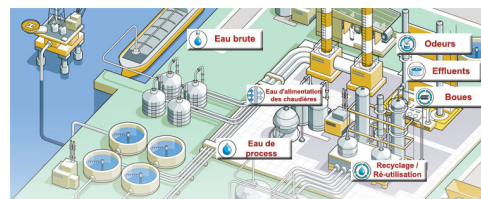
Electricity, electronics

RLC circuits, thermostats, regulation, refrigeration, computers, internet and telecommunications in general, photography and digital video



Chemistry

Chemical kinetics, engineering process, petroleum, distillation, petrochemical industry



Economics

Gain optimization, control of financial flux,
Market prevision

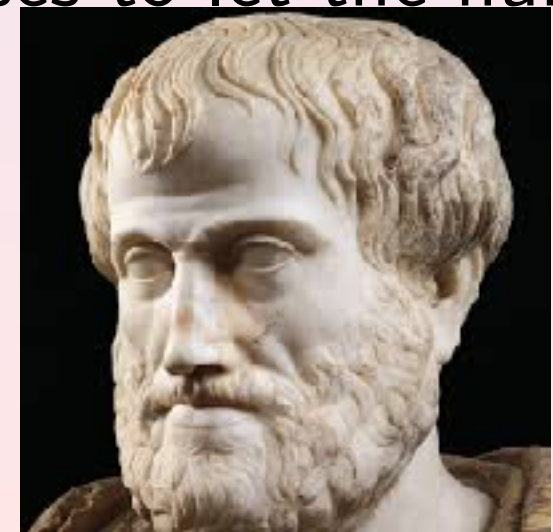


The origins

“... if every instrument could accomplish its own work, obeying or anticipating the will of others . . . if the shuttle weaved and the pick touched the lyre without a hand to guide them, chief workmen would not need servants, nor masters slaves.”

Book I, Chapter II, of the monograph “Politics” by [Aristotle \(384-322 B.C.\)](#).

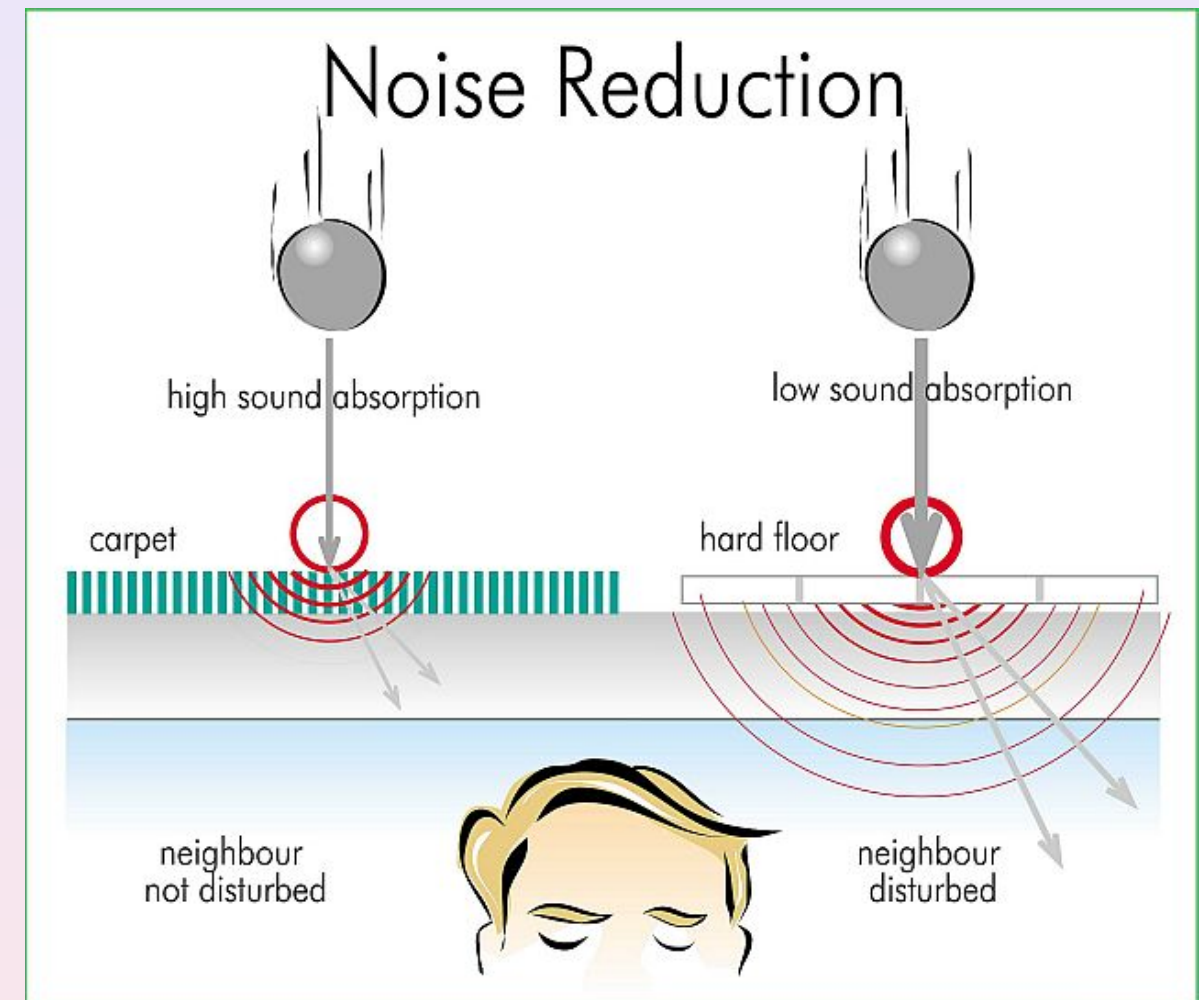
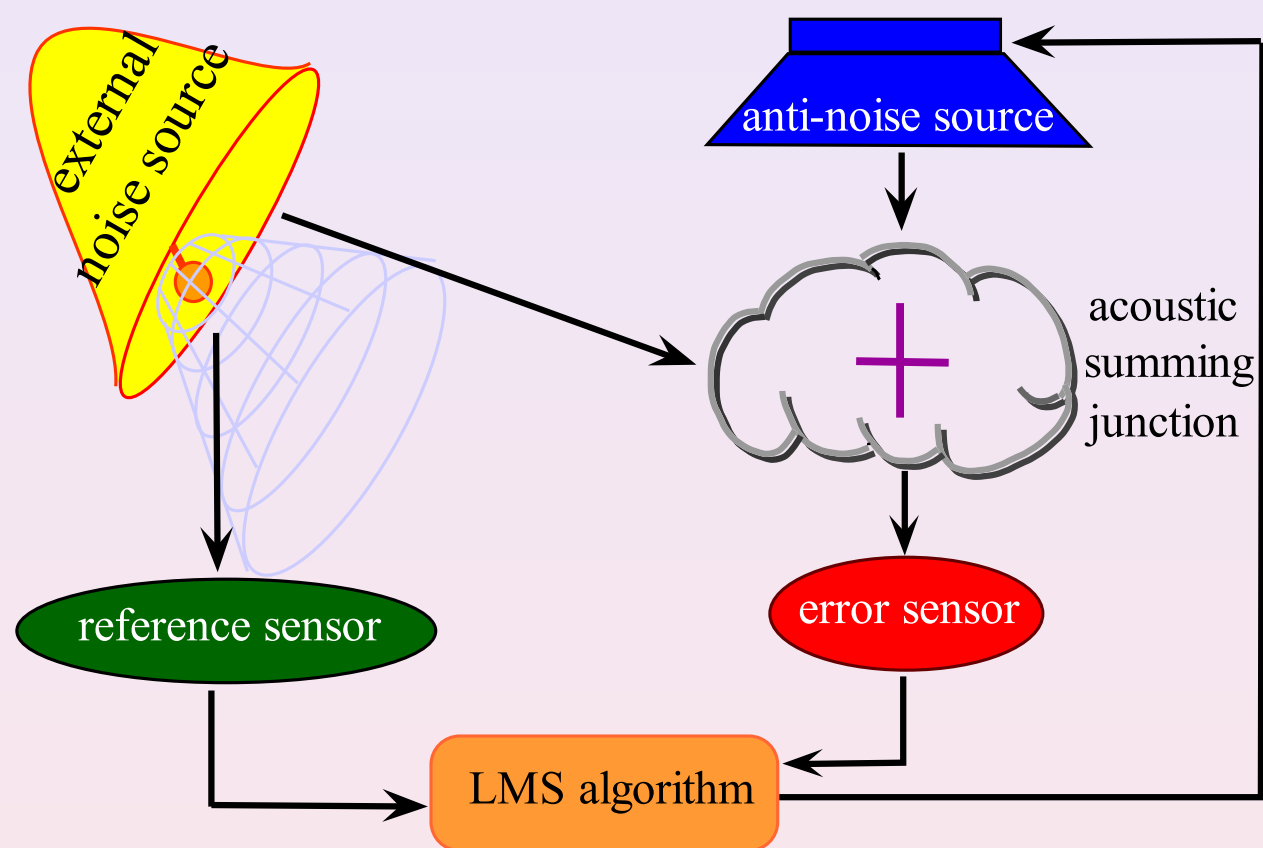
Main motivation: The need of automatizing processes to let the human being gain in liberty, freedom, and quality of life.





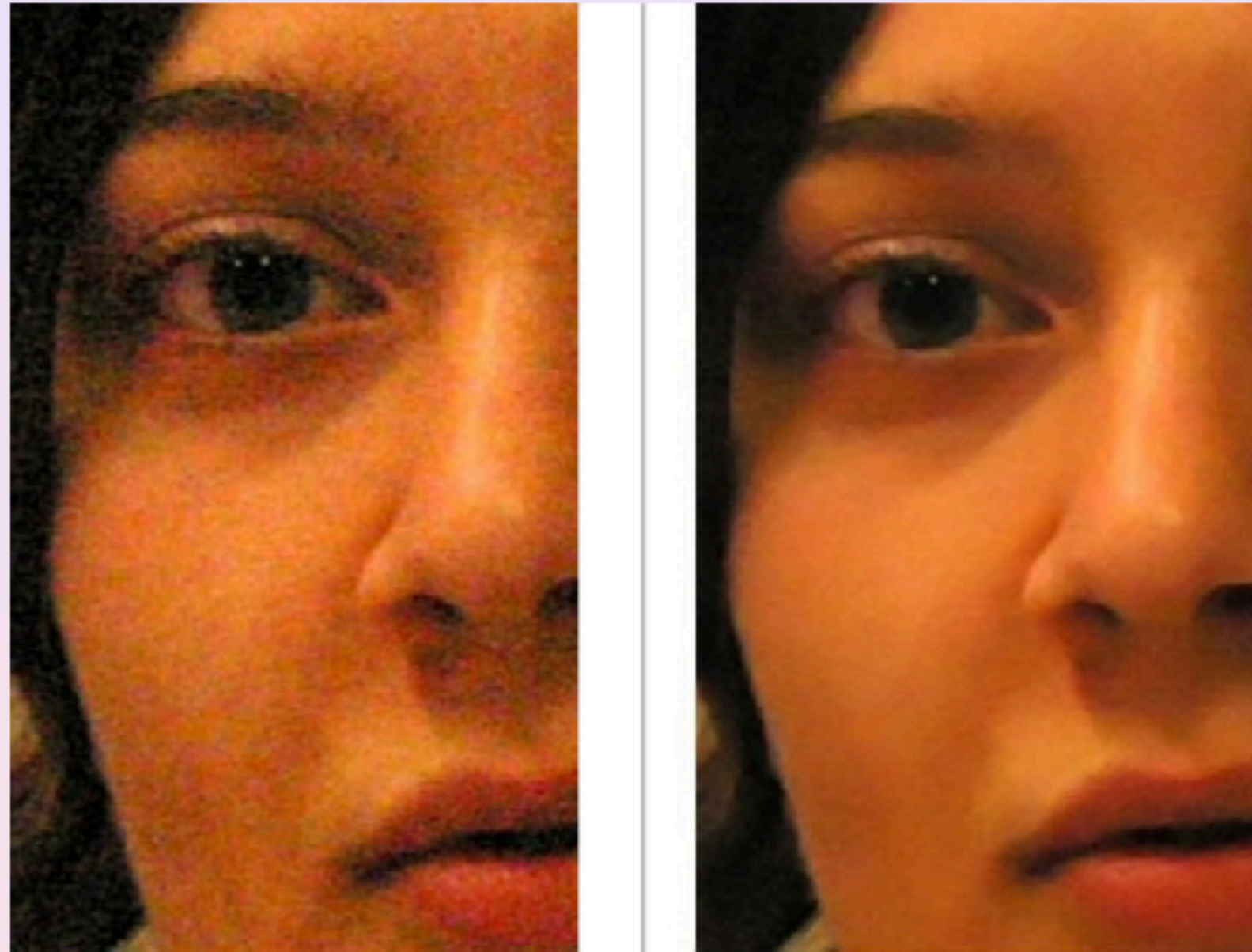
Control in an information rich World, SIAM, R. Murray Ed., 2003.

An example: noise reduction

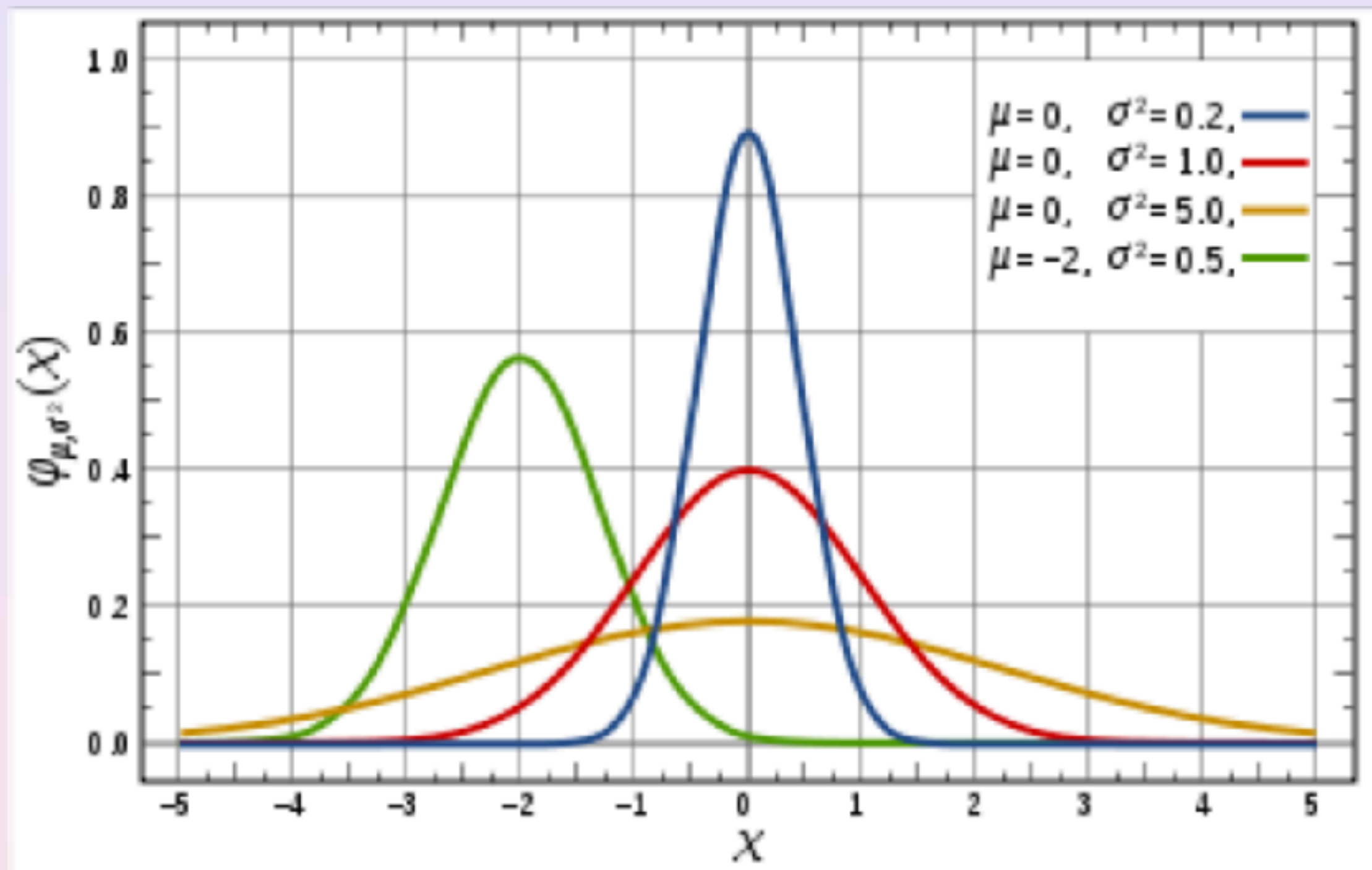


Acoustic noise reduction

Other applications of noise reduction



Gaussian filters



$$u(x) = [G(\cdot) \star f(\cdot)](x); \quad G(x) = (4\pi)^{-N/2} \exp(-|x|^2/4).$$

- The state equation

$$\dot{A}(y) = f(v). \quad (1)$$

- y is the state to be controlled.
- v is the control. It belongs to the set of admissible controls .
- Roughly speaking the goal is to drive the state y close to a desired state y_d :

$$y \sim y_d.$$

In this general functional setting many different mathematical models exist:

- Linear or nonlinear problems;
- Deterministic or stochastic models;
- Finite dimensional or infinite dimensional models;
- Ordinary Differential Equations (ODE) or Partial Differential Equations (PDE).

And, of course, when facing complex real life processes, often, hybrid models might also be needed.

Several kinds of different control problems may also fit in this frame depending on how the control objective is formulated:

- **Optimal control** (related with the **Calculus of Variations**)

$$\min_{v \in U_{ad}} \|y - y_d\|^2.$$

- **Controllability**: Drive exactly the state y to the prescribed one y_d .

This is a more **dynamical notion**.

Several relaxed versions also arise: approximate controllability.

- **Stabilization** or **feedback control**. (real time control)

$$v = F(y); \quad A(y) = f(F(y)).$$

The concept of **feedback**. Inspired in the capacity of biological systems to self-regulate their activities.

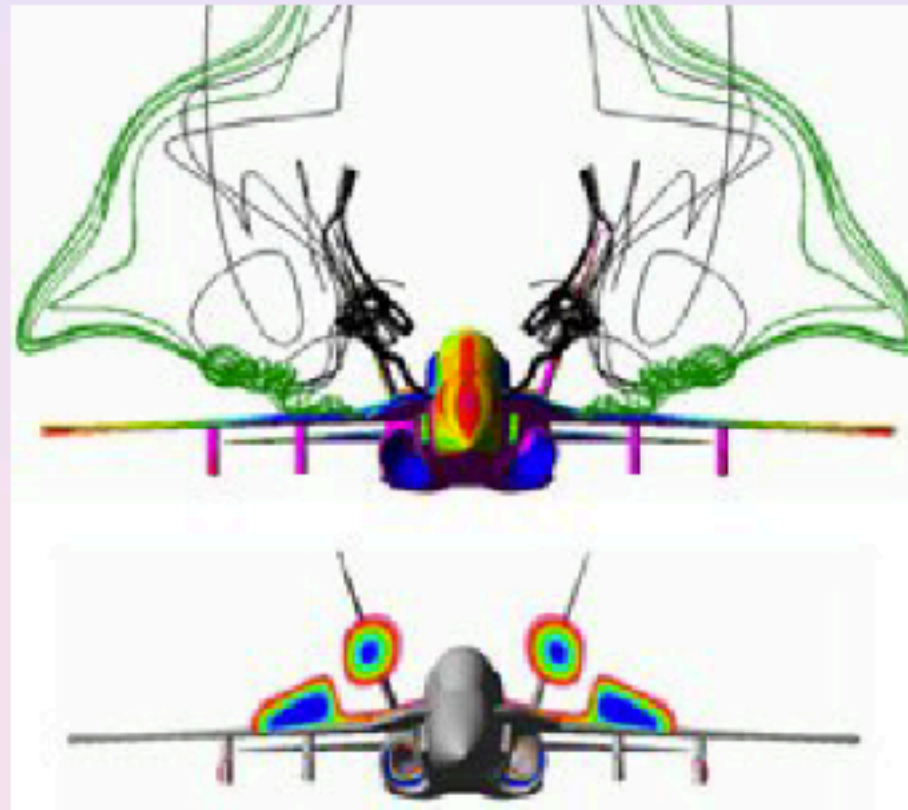
Incorporated to Control Engineering in the twenties by the engineers of the “Bell Telephone Laboratory” but, at that time, it was already recognized and consolidated in other areas, such as Political Economics.

Feedback process: the one in which the state of the system determines the way the control has to be exerted in real time
Nowadays, feedback processes are ubiquitous in applications to Engineering, Economy also in Biology, Psychology, etc.

Cause-effect principle → **Cause-effect-cause principle**.

Some examples

- The thermostat;
- The control of aircrafts in flight or vehicles in motion:



The need of **fluctuations**.

“It is a curious fact that, while political economists recognize that for the proper action of the law of supply and demand there must be fluctuations, it has not generally been recognized by mechanicians in this matter of the steam engine governor. The aim of the mechanical engineers, as is that of the political economist, should be not to do away with these fluctuations all together (for then he does away with the principles of self-regulation), but to diminish them as much as possible, still leaving them large enough to have sufficient regulating power.”

H.R. Hall, *Governors and Governing Mechanisms*, The Technical Publishing Co., 2nd ed., Manchester 1907.

An example: Lagrange multipliers.

$$\min_{g(x)=c} f(x).$$

The answer: critical points x are those for which

$$\nabla f(x) = \lambda \nabla g(x)$$

for some real λ .

This is so because $\nabla g(x)$ is the normal to the level set in which minimization occurs. A necessary condition for the point x to be critical is that $\nabla f(x)$ points in this normal direction. Otherwise, if $\nabla f(x)$ had a nontrivial projection over the level set $g(x) = c$ there would necessarily exist a better choice of x for which $f(x)$ would be even smaller.

Automatic control. The number of applications rapidly increased in the thirties covering different areas like amplifiers in telecommunications, distribution systems in electrical plants, stabilization of aeroplanes, electrical mechanisms in paper production, petroleum and steel industry,...

By that time there were two clear and distinct approaches:

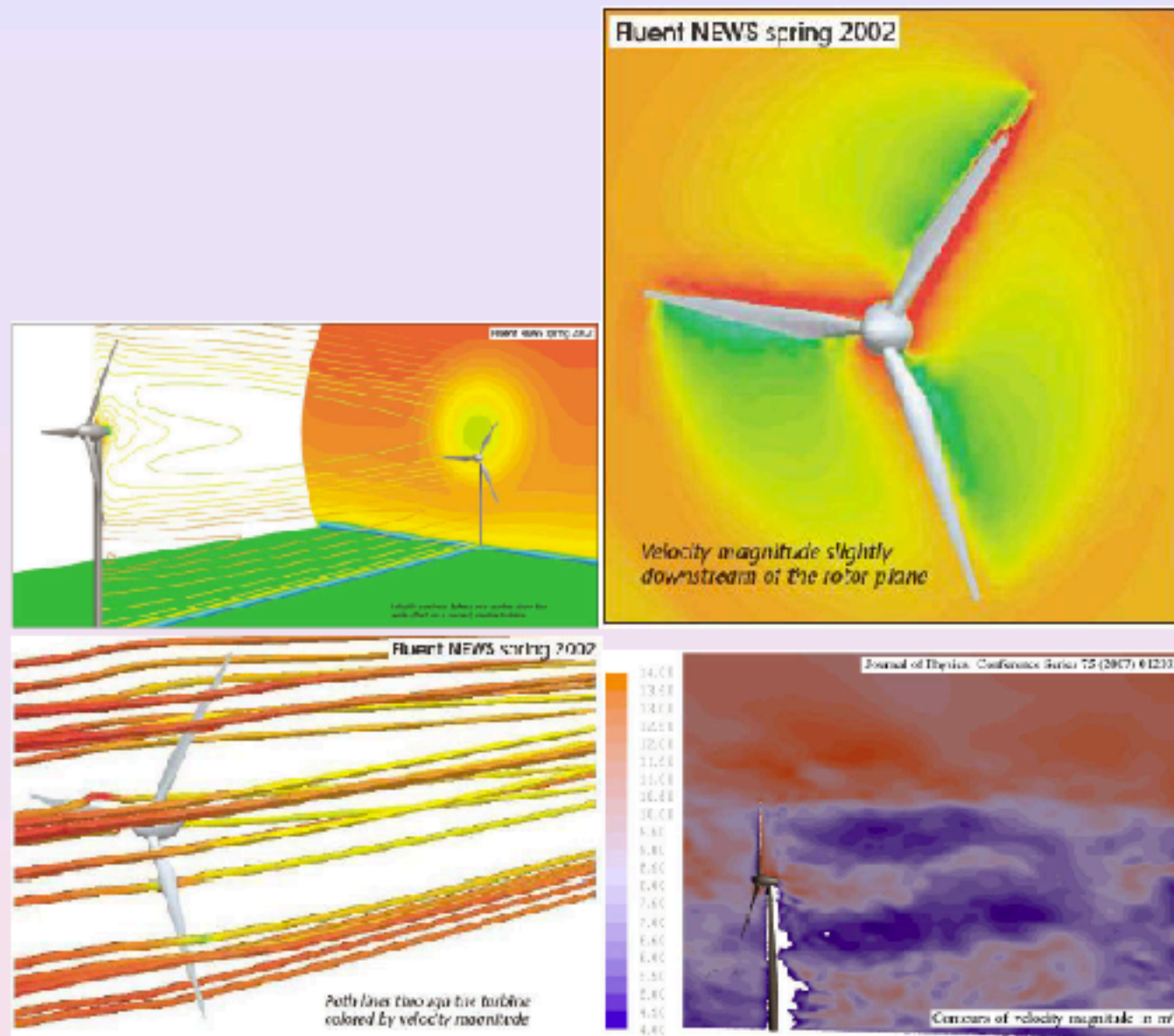
- **State space approach**, based on modelling by means of Ordinary Differential equations (ODE);
- **The frequency domain** approach, based in the Fourier representation of signals.

PHYSICAL SPACE \equiv FREQUENCY SPACE

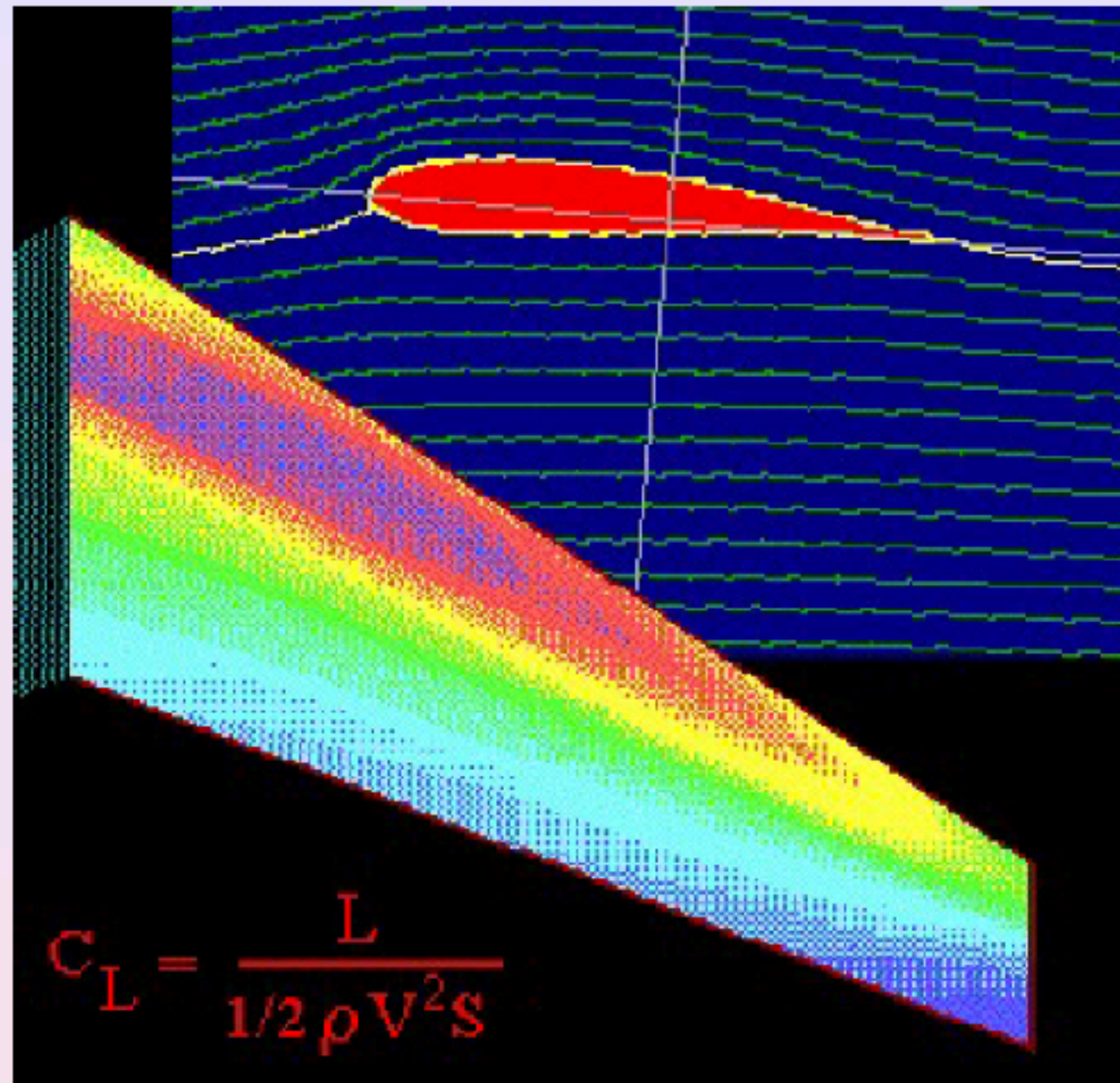
But after the second world war it was discovered that most physical systems were **nonlinear** and **nondeterministic**.



Aerospace industry



Eolic energy generation



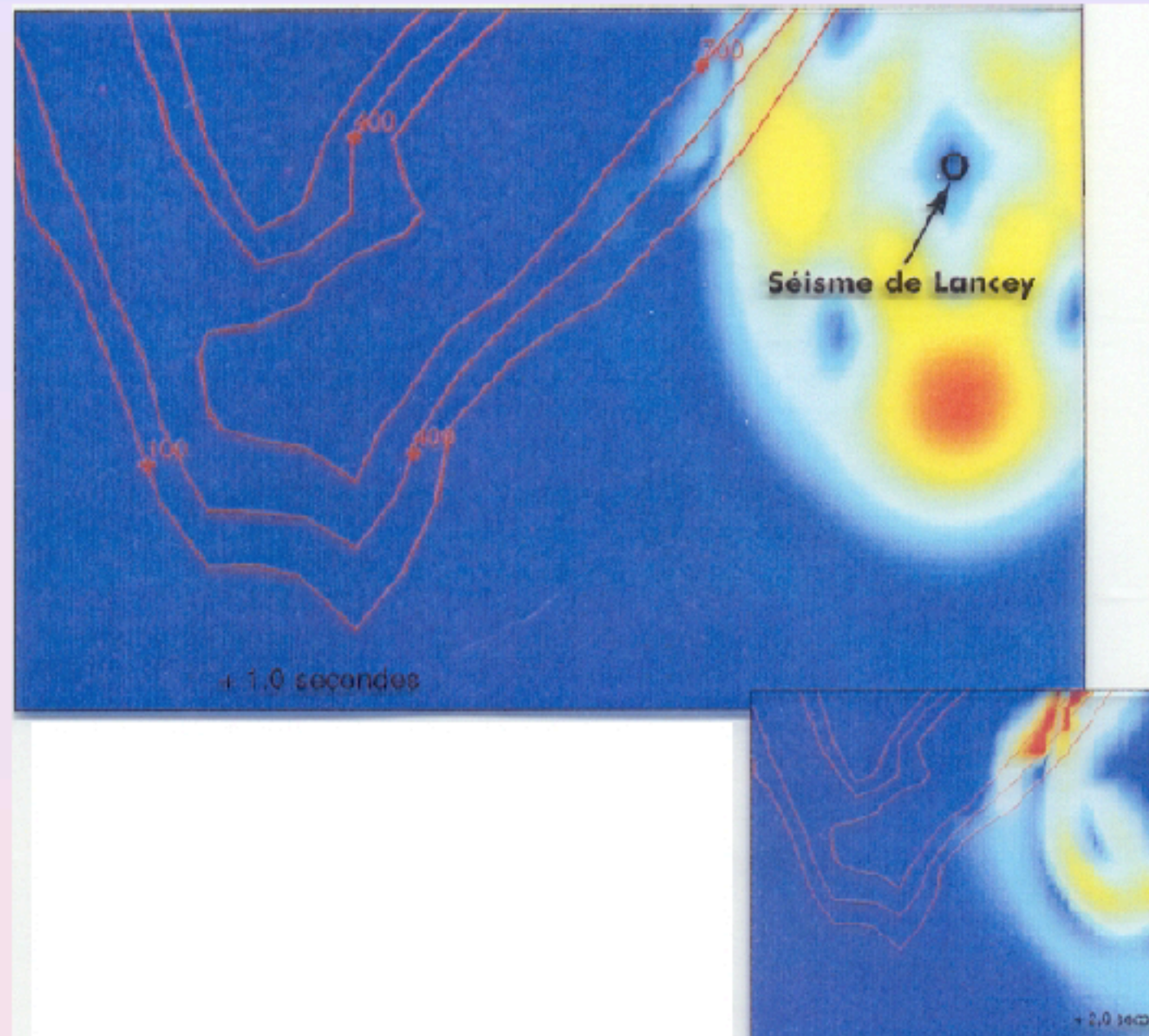
Optimal shape design in aerodynamics

- Flexible structures.



SIAM Report on "Future Directions in Control Theory. A Mathematical Perspective", W. H. Fleming et al., 1988.

- Seismic waves, earthquakes.

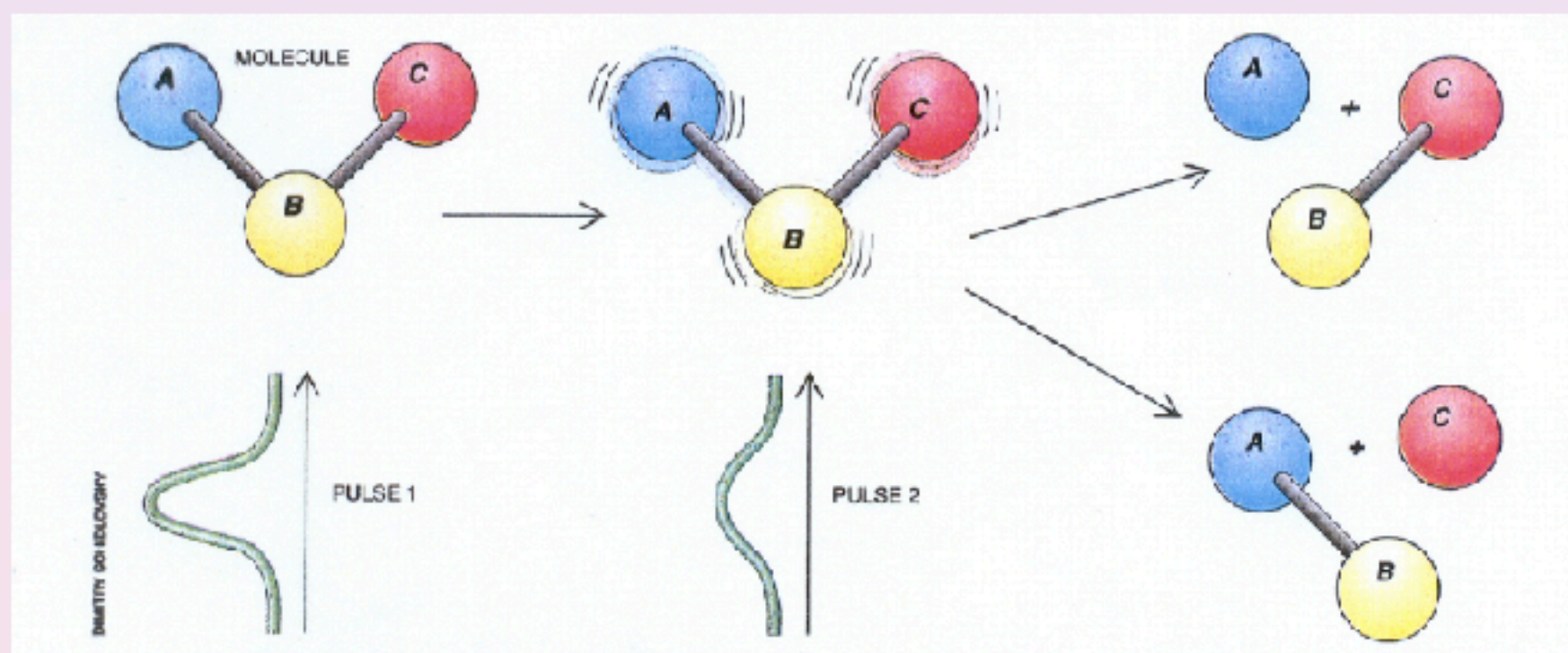


F. Cotton, P.-Y. Bard, C. Berge et D. Hatzfeld, Qu'est-ce qui fait vibrer Grenoble?, La Recherche, 320, Mai, 1999, 39-43.

- Quantum control and Computing.

Laser control in Quantum mechanical and molecular systems to design **coherent vibrational states**.

In this case the fundamental equation is the Schrödinger one. Most of the theory we shall develop here applies in this case too. The **Schrödinger equation** may be viewed as a **wave equation** with infinite speed of propagation.



P. Brumer and M. Shapiro, Laser Control of Chemical reactions, Scientific American, March, 1995, pp.34-39.

- Irrigation systems, ancient Mesopotamia, 2000 BC.
- Harpenodaptai, ancient Egypt, the string stretchers.
 - Primal: The minimal distance between two points is given by the straight line.
 - Dual: The maximal distance between the extremes of a cord is obtained when the cord is along a straight line.

In mathematical terms, things are not easy:

To minimize the functional

$$\int_0^1 ||x'(t)|| dt$$

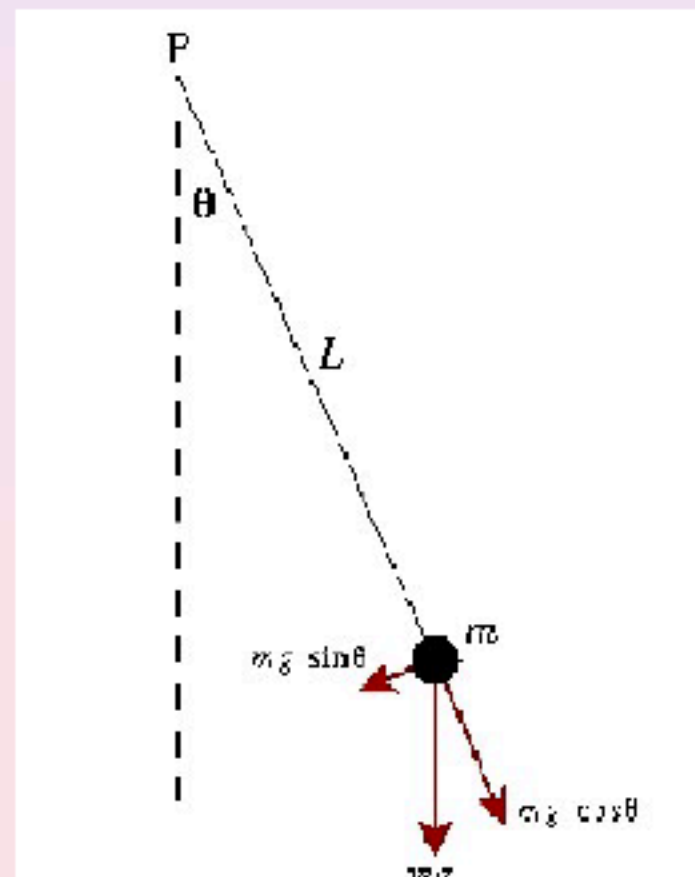
among the set of parametrized curves $x : [0, 1] \rightarrow \mathbf{R}^d$, such that $x(0) = A$ and $x(1) = B$.

We easily end up working in the BV class of functions of bounded variation, out of the most natural and simple context of Hilbert spaces.

Roman aqueducts. Systems of water transportation endowed with valves and regulators.



The pendulum. The works of Ch. Huygens and R. Hooke, in the end of the XVII century, the goal being measuring in a precise way location and time, so precious in navigation.



The first mathematical rigorous analysis of the stability properties of the steam engine was done by Lord J. C. Maxwell, in 1868.

The explanation of some erratic behaviors was explained. Until then it was not well understood why apparently more elaborated and perfect regulators could have a bad behavior.

The reason is now referred to as **the overdamping phenomenon**.

Consider the equation of the **pendulum**:

$$x'' + x = 0.$$

This describes a pure conservative dynamics: the energy

$$e(t) = \frac{1}{2}[x^2(t) + |x'(t)|^2]$$

is constant in time.

Let us now consider the dynamics of the pendulum in presence of a **friction** term:

$$x'' + x = -kx',$$

k being a positive constant $k > 0$.

The energy decays exponentially. But the decay rate does not necessarily increase with the damping parameter k .

Indeed, computed the eigenvalues of the characteristic equation one finds:

$$\lambda_{\pm} = [-k \pm \sqrt{k^2 - 4}]/2.$$

It is easy to see that λ_+ increases as $k > 2$ increases.

Indeed,

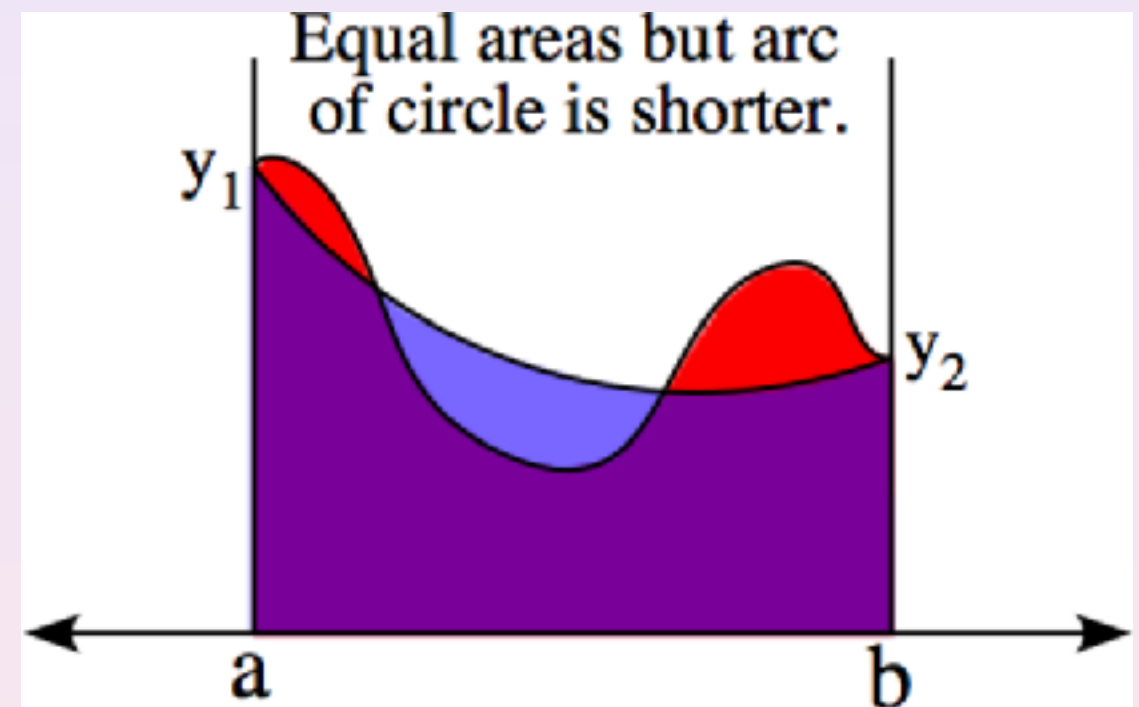
$$\lambda_+ = \frac{-k^2 + k^2 - 4}{k + \sqrt{k^2 - 4}} = \frac{-4}{k + \sqrt{k^2 - 4}}.$$

This shows that $\lambda_+ \rightarrow 0$ when $k \rightarrow +\infty$, so, the decay of the system gets worse when k is very large.

This confirms the prediction that optimal controls and strategies are often complex and that they do not necessarily obey to the very first intuition.

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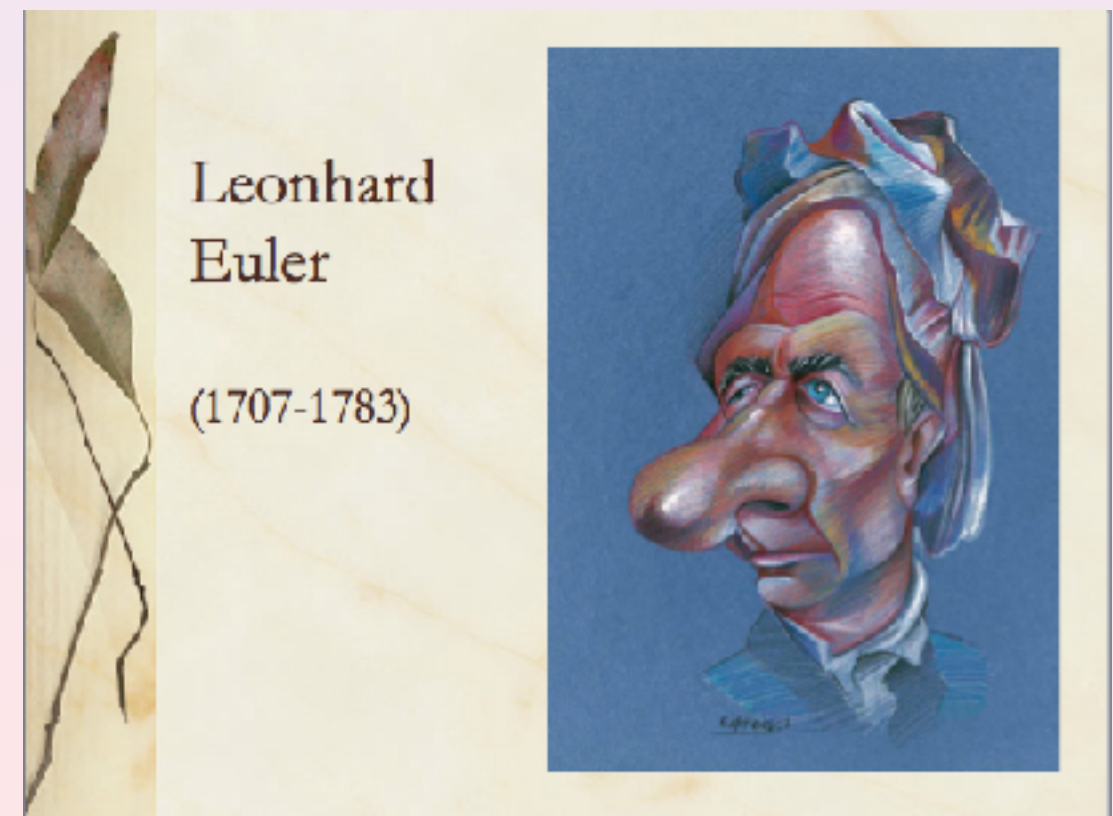
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$$e^{i\pi} + 1 = 0$$

Leonhard Euler (1707-1783)

For since the fabric of the universe is most perfect and the work of a most wise creator, nothing at all takes place in the universe in which some rule of the maximum or minimum does not appear.



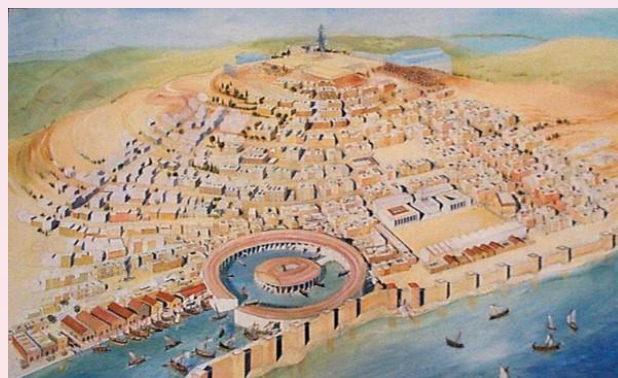
The Calculus of Variations = The search of minimality : Isoperimetric inequalities

The isoperimetric inequality states, for the length L of a closed curve and the area A of the planar region that it encloses, that

$$4\pi A \leq L^2,$$

and that equality holds if and only if the curve is a circle.

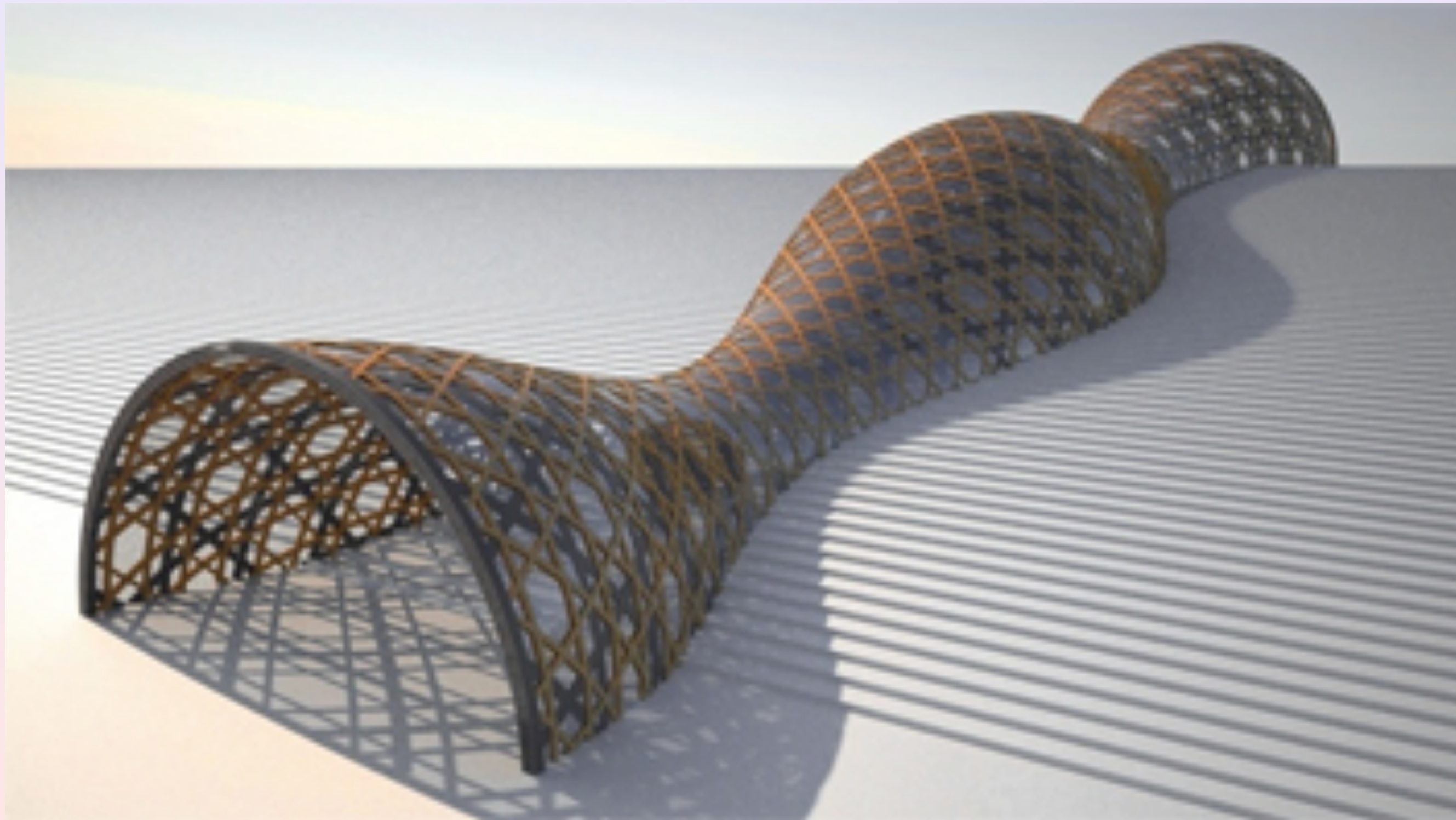
Dido's problem; named after Dido, the legendary founder and first queen of Carthage.



Carthage & Cologne

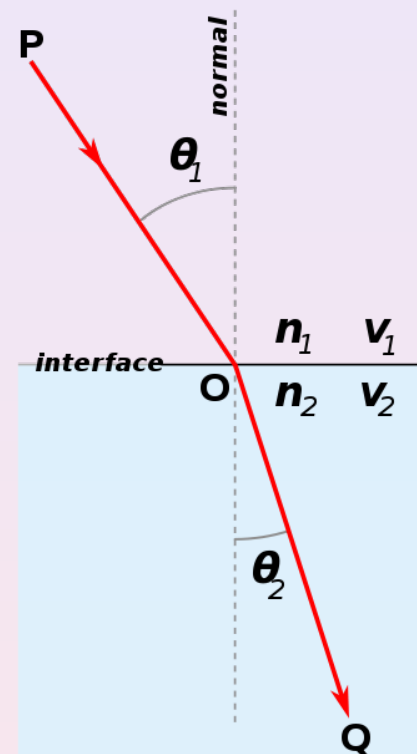
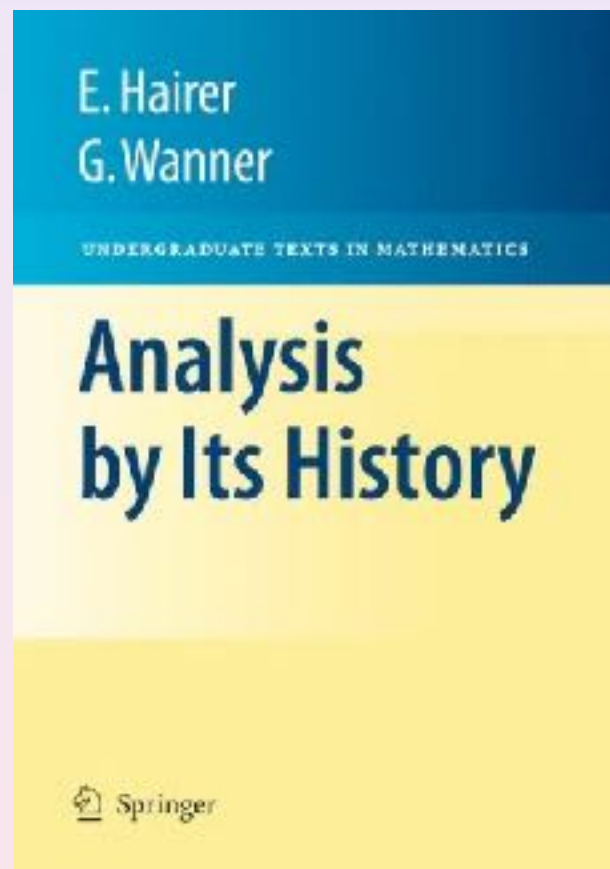
Geodesic curves

The curve of shortest length, **geodesic**, connecting two points.



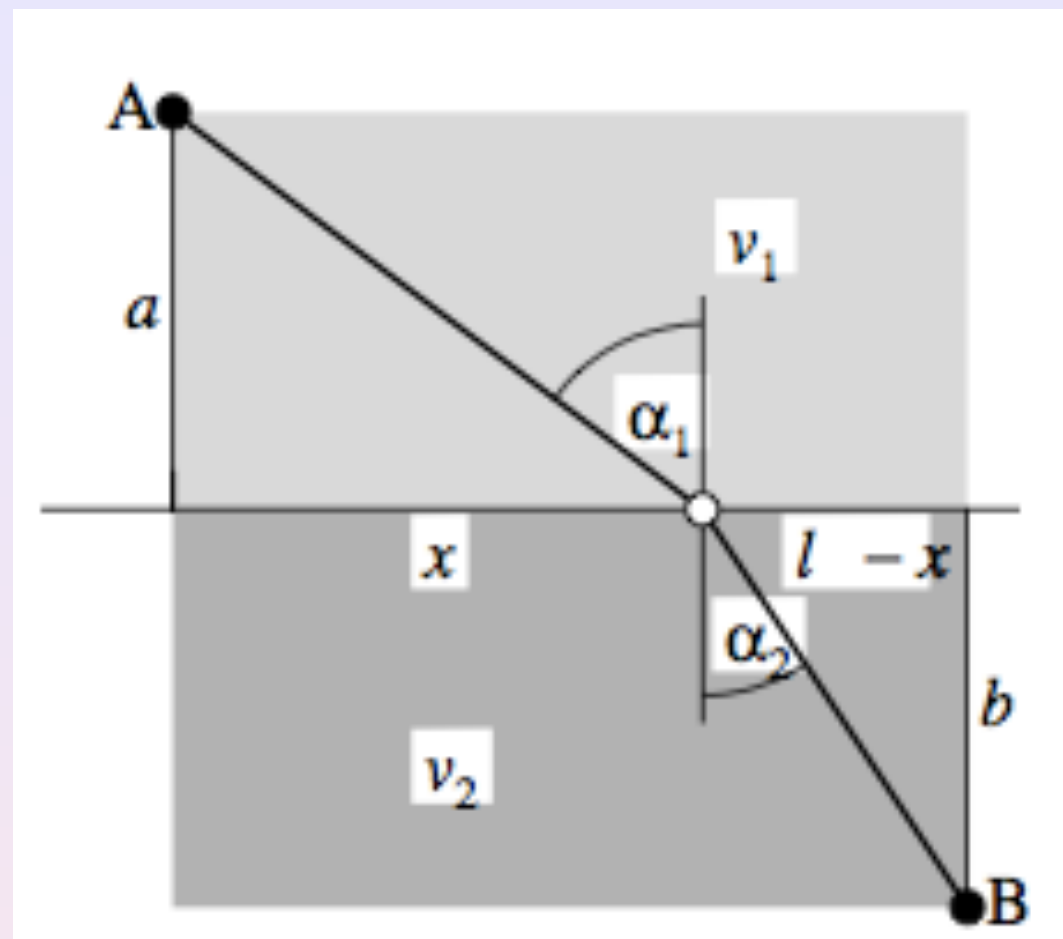
Fermat's principle/Snellious law

Fermat's principle: light follows the path of shortest optical length connecting two points, where the optical length depends upon the material of the medium.



$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2} = \frac{n_2}{n_1}.$$

Named after the dutch astronomer Willebrord Snellius (1580 - 1626).
Pierre de Fermat (1601 - 1665).



To find x s. t.

$$T = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (l - x)^2}}{v_2}.$$

Fermat found the problem too difficult for an analytical treatment (**I admit that this problem is not one of the easiest**). The computations were then proudly performed by Leibniz (1684)

$$T' = \frac{1}{v_1} \frac{2x}{2\sqrt{a^2 + x^2}} - \frac{1}{v_2} \frac{2(\ell - x)}{2\sqrt{b^2 + (\ell - x)^2}}.$$

Observing that $\sin(\alpha_1) = x/\sqrt{a^2 + x^2}$; $\sin(\alpha_2) = (\ell - x)/\sqrt{b^2 + (\ell - x)^2}$ we see that this derivative vanishes whenever

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{v_1}{v_2}.$$

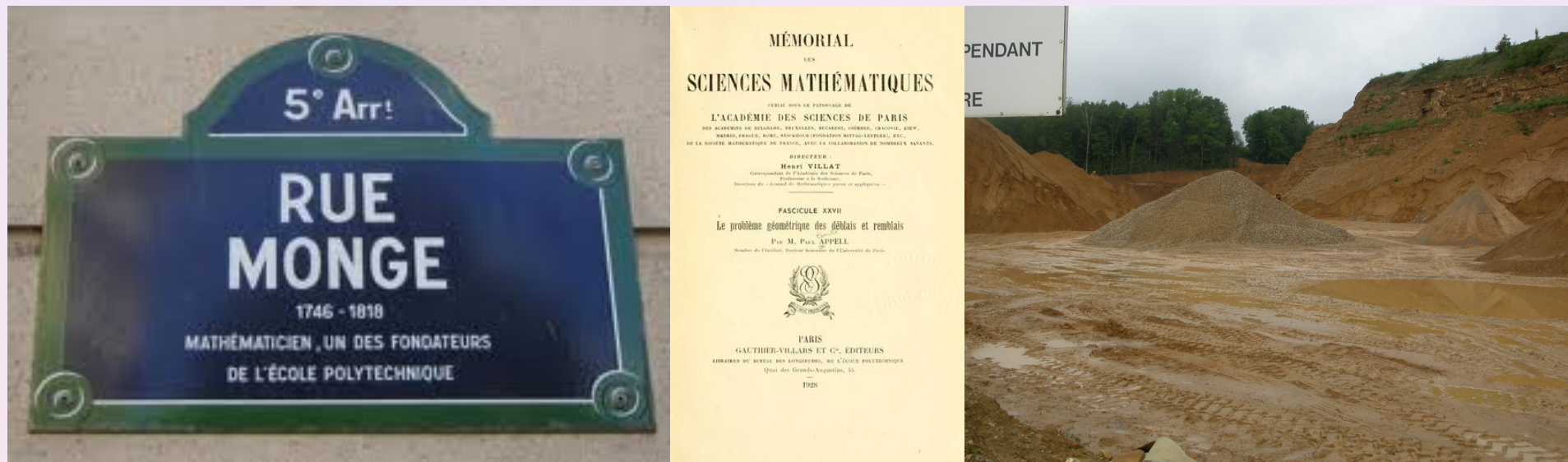
Furthermore:

$$T'' = \frac{1}{v_1} \frac{a^2}{(a^2 + x^2)^{3/2}} + \frac{1}{v_2} \frac{b^2}{(b^2 + (\ell - x)^2)^{3/2}} > 0,$$

showing that the critical point is the minimizer.

Optimal transport

In mathematics and economics, transportation theory refers to the study of optimal transportation and allocation of resources. The problem was formalized by the French mathematician **Gaspard Monge** in 1781 (“Sur la théorie des déblais et des remblais” (Mém. de l’Acad. de Paris, 1781))



But the origins of the potential applications of the idea of optimal transport and geodesic paths goes back to the ancient Egypt where the “harpenodaptai” had as main task drawing long straight lines on the sand of the desert.

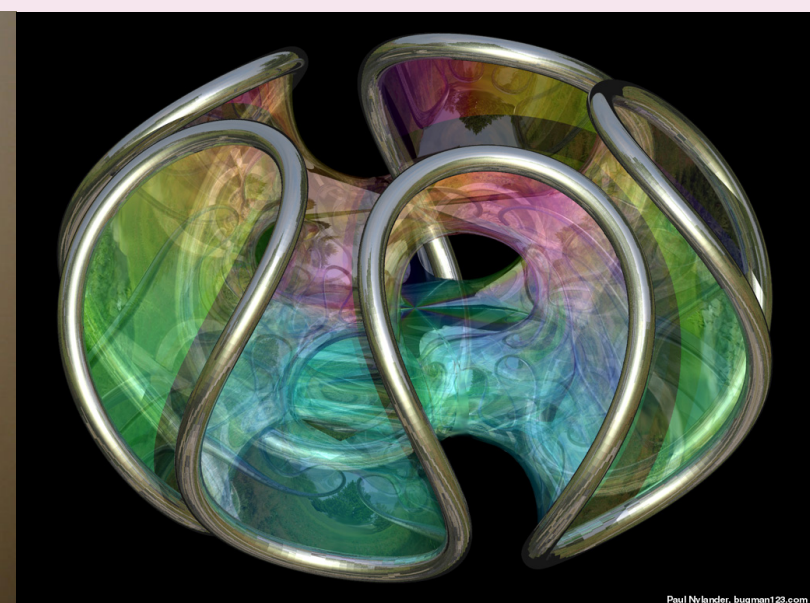
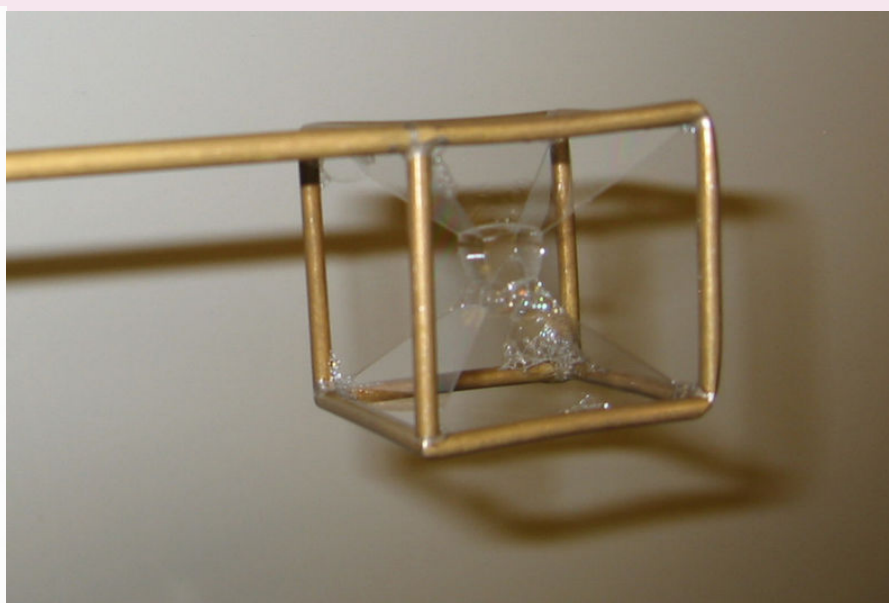
Minimal surfaces

Minimal surfaces are defined as surfaces with zero mean curvature. Finding a minimal surface of a boundary with specified constraints is a problem in the Calculus of Variations and is sometimes known as **Plateau's problem**.

Physical models of area-minimizing minimal surfaces can be made by dipping a wire frame into a soap solution, forming a soap film, which is a minimal surface whose boundary is the wire frame.

Enneper's surface:

$$x = u(1 - u^2/3 + v^2)/3; y = -v(1 - v^2/3 + u^2)/3; z = (u^2 - v^2)/3.$$

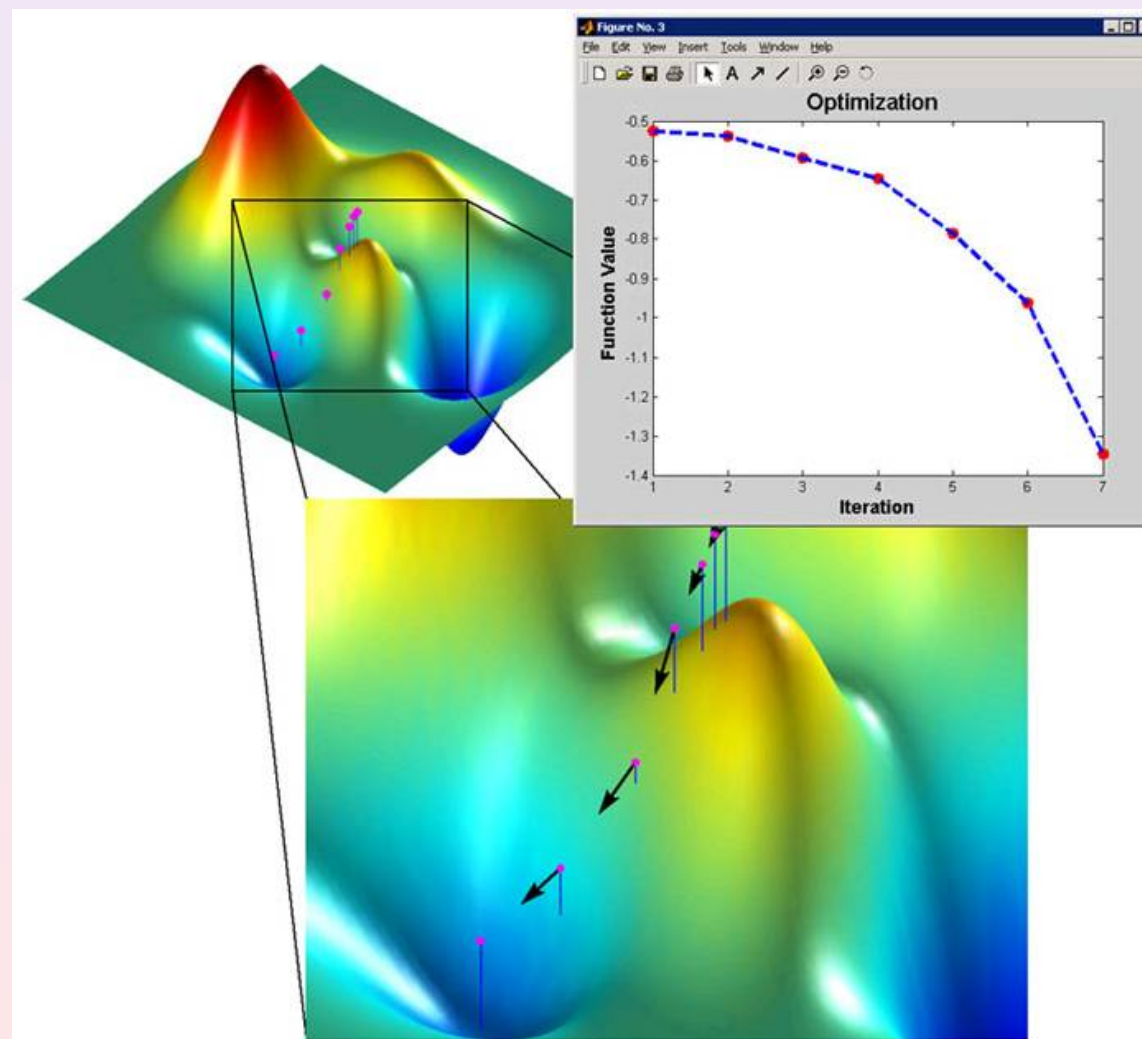


Computational minimization

$$J(u^*) = \min_{u \in \mathcal{U}} J(u).$$

Gradient methods:

$$u_{k+1} = u_k - \rho \nabla J(u_k).$$



Montecarlo methods

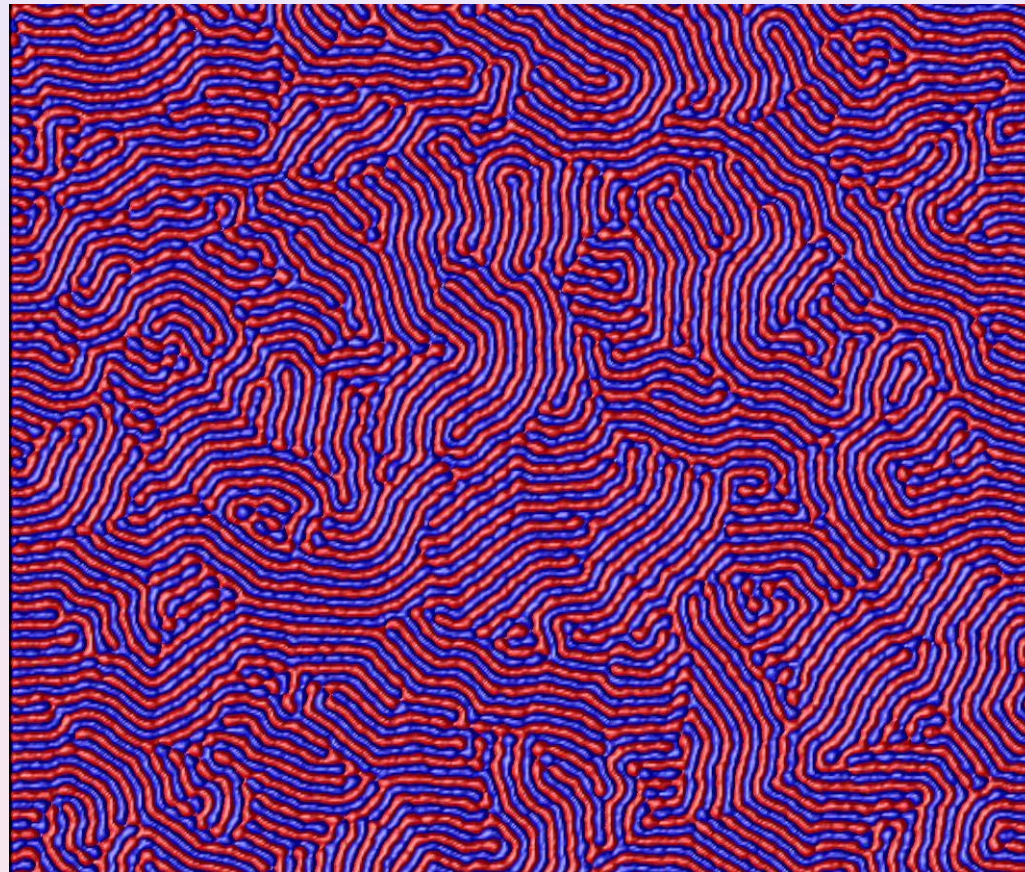
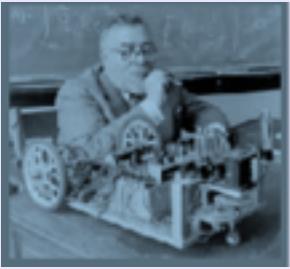


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“Cybernétique” was proposed by the French physicist **A.-M. Ampère** in the XIX Century to design the nonexistent science of process controlling. This was quickly forgotten until 1948, when **Norbert Wiener** (1894–1964) chose “**Cybernetics**” as the title of his famous book.

Wiener defined Cybernetics as “ **the science of control and communication in animals and machines**” .

In this way, he established the connection between Control Theory and Physiology and anticipated that, in a desirable future, engines would obey and imitate human beings.



Robotic arm





Let $n, m \in \mathbb{N}^*$ and $T > 0$ and consider the following linear finite-dimensional system

$$x'(t) = Ax(t) + Bu(t), \quad t \in (0, T); \quad x(0) = x^0. \quad (1)$$

In (1), A is a $n \times n$ real matrix, B is of dimensions $n \times m$ and x^0 is the initial state of the system in \mathbb{R}^n . The function $x : [0, T] \rightarrow \mathbb{R}^n$ represents the *state* and $u : [0, T] \rightarrow \mathbb{R}^m$ the *control*.

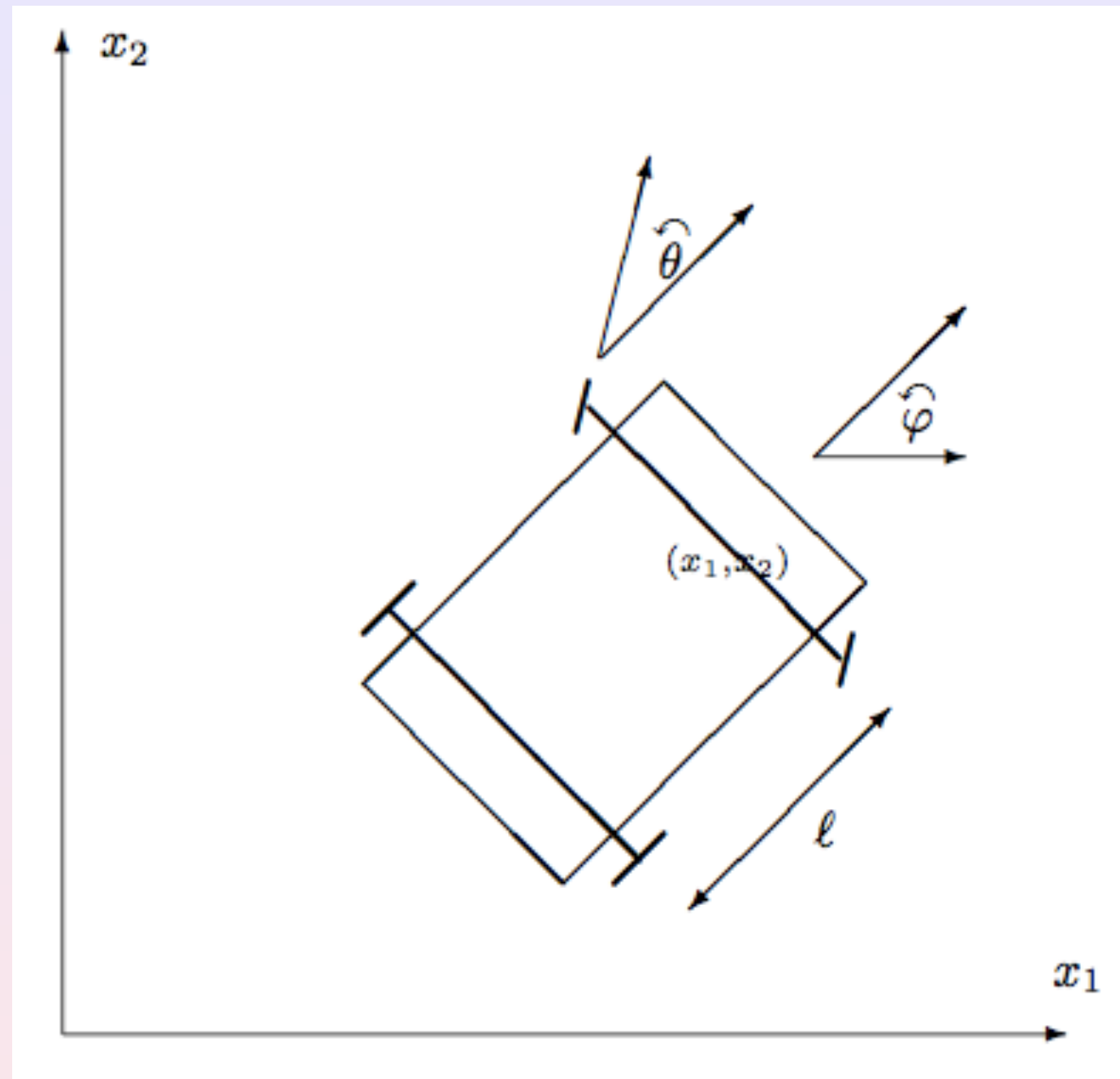
¿Can we control the state x of n components with only m controls, even if $n \gg m$?

Theorem

(1958, Rudolf Emil Kálmán (1930–2016)) System (1) is controllable iff

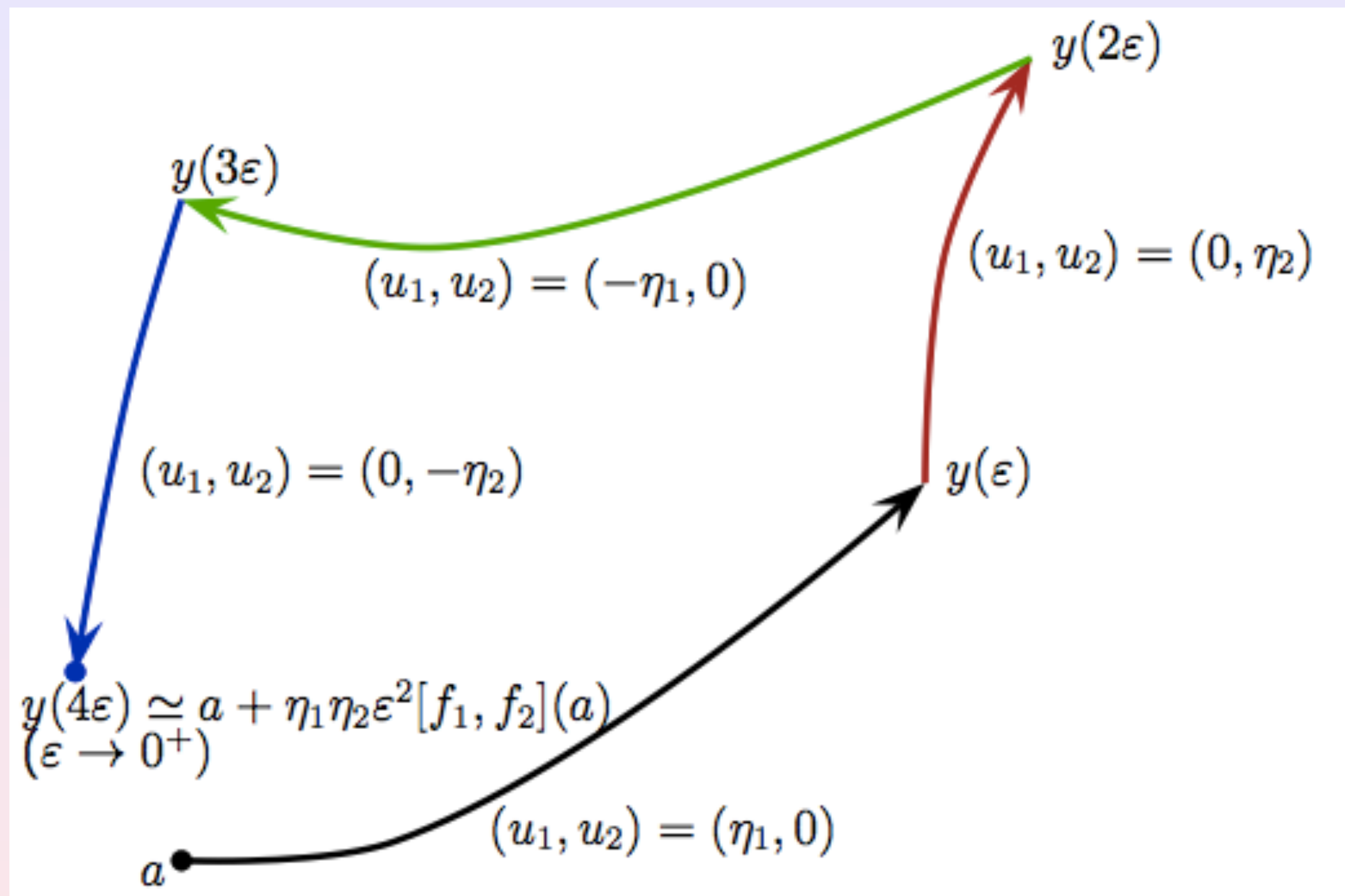
$$\text{rank}[B, AB, \dots, A^{n-1}B] = n.$$

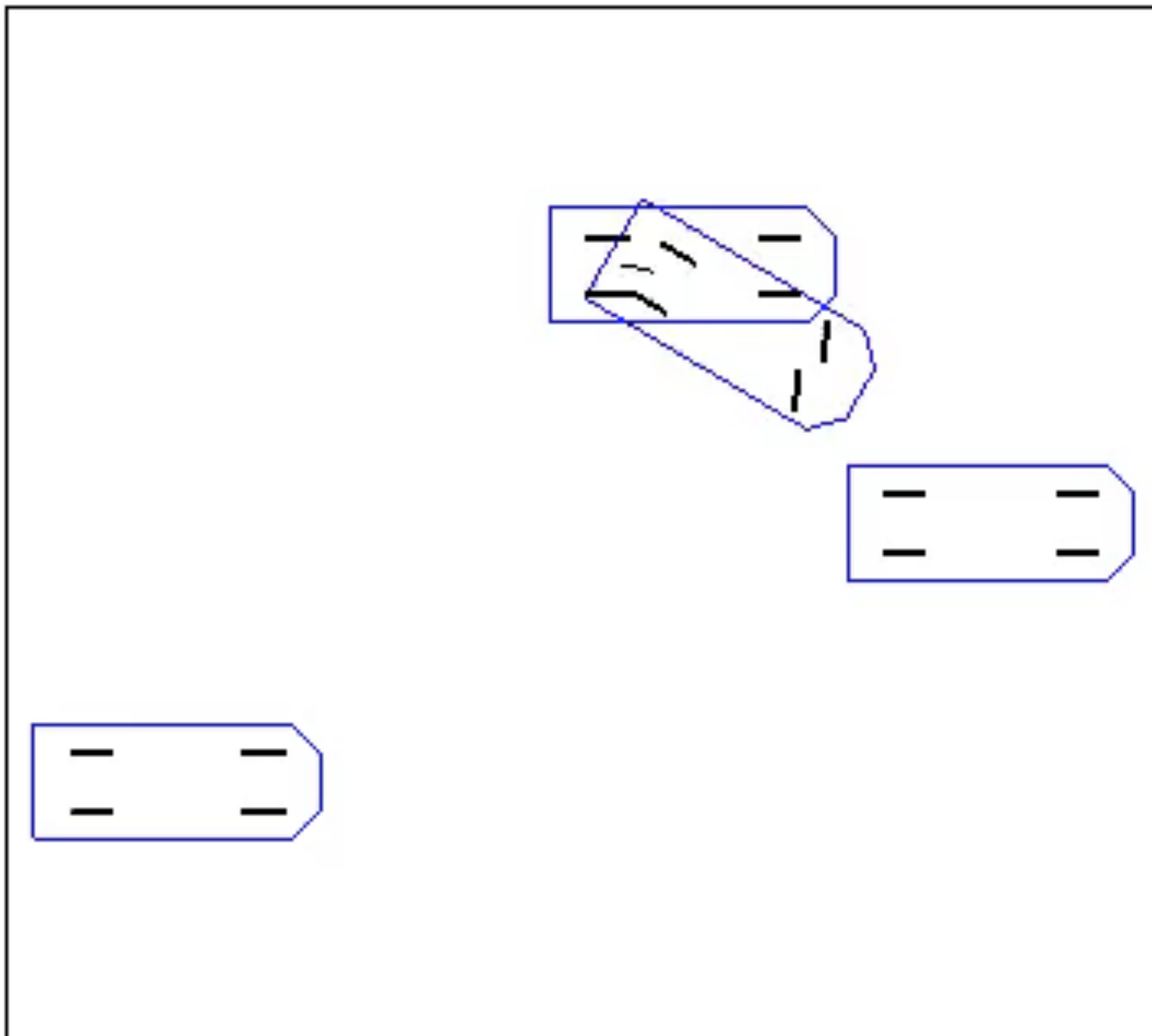
An example: Nelson's car.



Two controls suffice to control a four-dimensional dynamical system.

E. Sontag, *Mathematical control theory*, 2nd ed., Texts in Applied Mathematics, vol. 6, Springer-Verlag, New York, 1998.





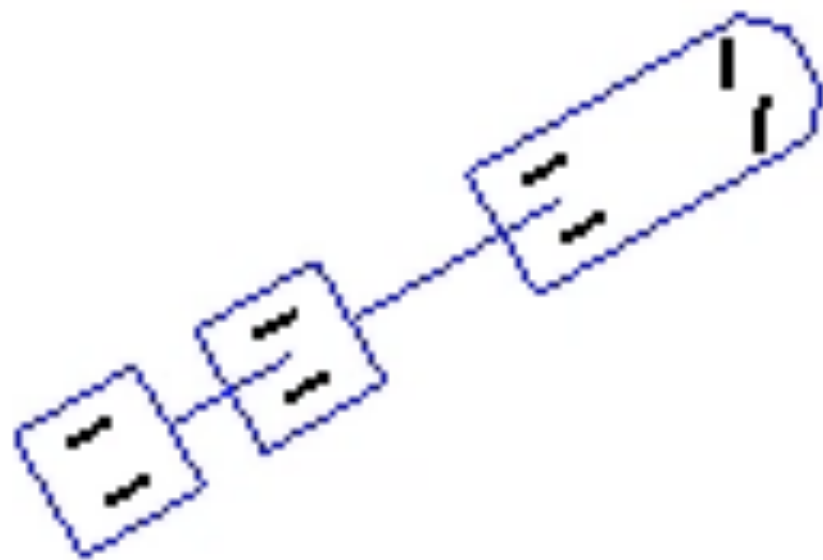




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The optimal pancake

MailOnline

The perfect pancake? Easy, just follow this formula ... $100 - [10L - 7F + C(k - C) + T(m - T)] / (S - E)$

By [Daily Mail Reporter](#)

Last updated at 9:49 AM on 24th February 2009

With Shrove Tuesday tomorrow it was perhaps inevitable that an eager scientist would apply their skills to creating the perfect pancake.

Maths expert Dr Ruth Fairclough stepped up to the challenge, unveiling a complex algebra formula to help chefs nail the dish on the day.

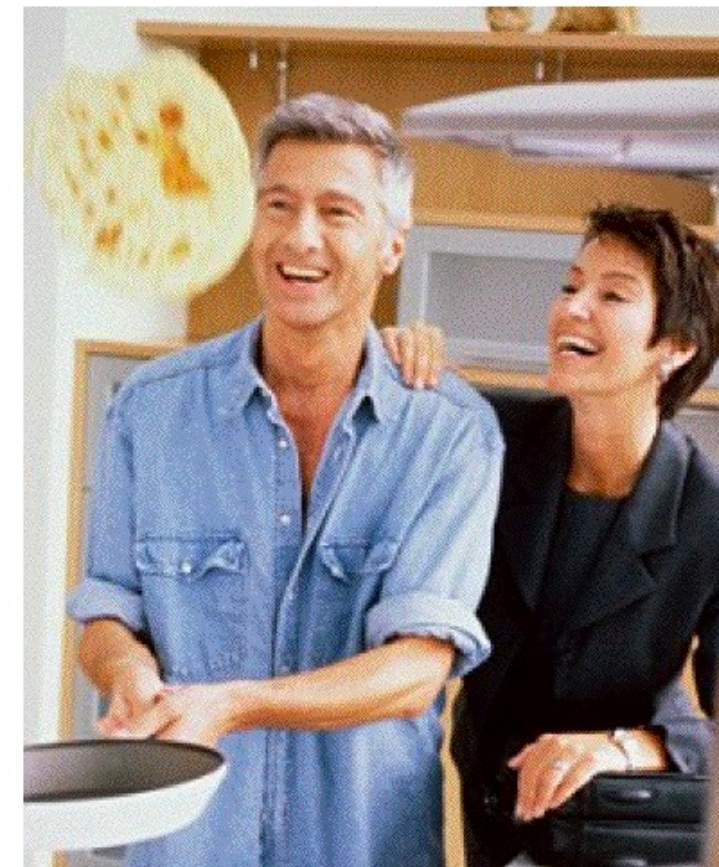
The 34-year-old senior lecturer of mathematics and statistics worked out the food formula because her two daughters loved eating pancakes so much.

Dr Ruth, who teaches at Wolverhampton University found that $100 - [10L - 7F + C(k - C) + T(m - T)] / (S - E)$ created the tastiest snack.

In the complex formula L represents the number of lumps in the batter and C equals its consistency.

The letter F stands for the flipping score, k is the ideal consistency and T is the temperature of the pan.

Ideal temp of pan is represented by m, S is the length of time the batter stands before cooking and E is the length of time the cooked pancake sits before being eaten.





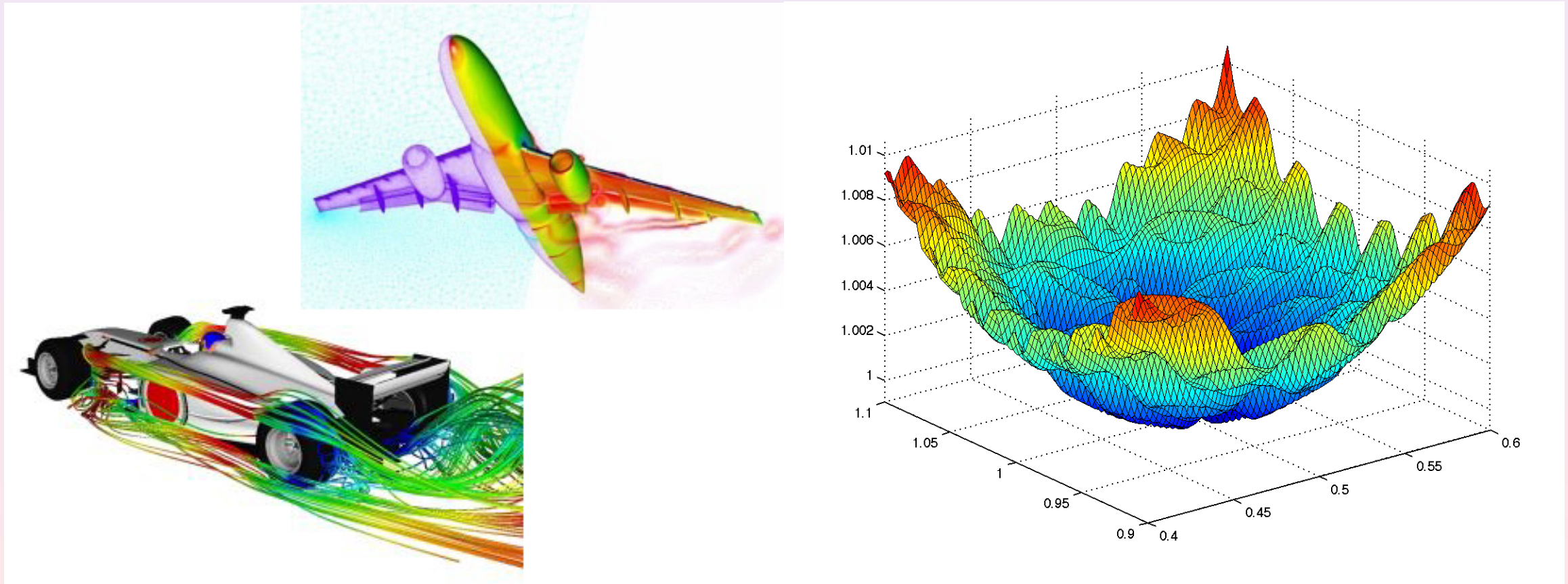
Optimal shape design in aeronautics.

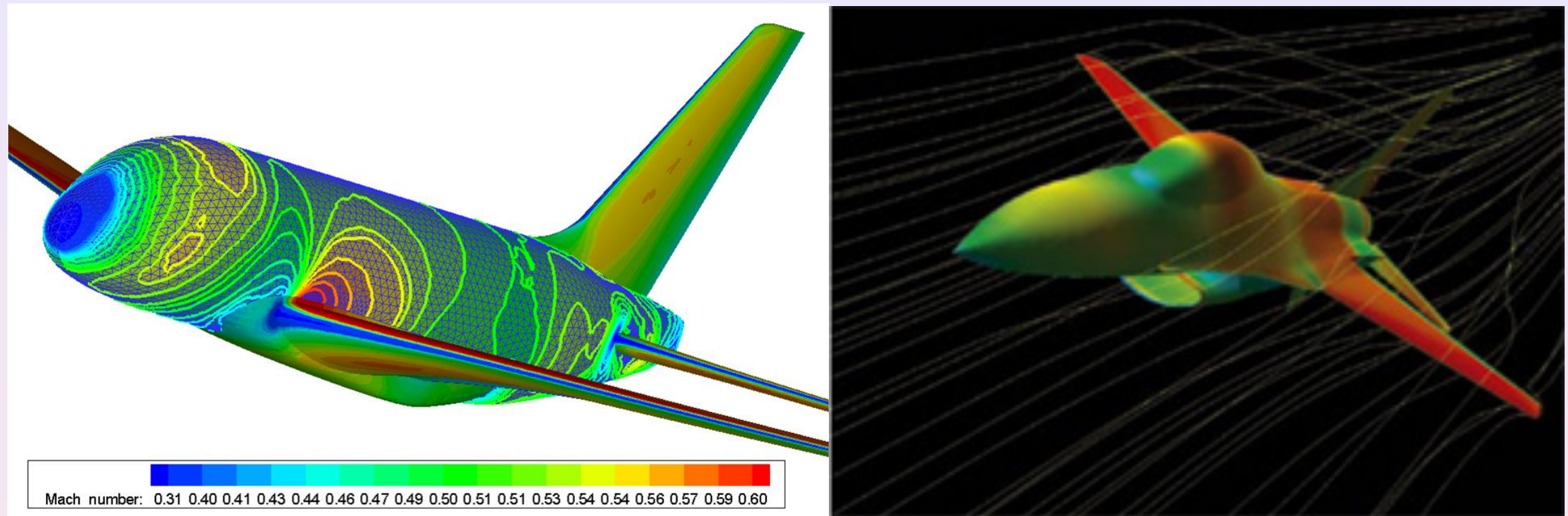
- **Objective:** To modify the shape of the airplane so to improve its efficiency, security, reduce noise, energy consumption, reduce drag, augment lift,...
- **Point of view:** That of the wind tunnel. The airplane is fixed while air is flowing around.
- **Variations:** When modifying the shape of the airplane, the way air is flowing around is modified, and the pressure field it applies into the airplane as well. The aerodynamical properties of the airplane are modified.



Tools

- **Computational fluid mechanics:** It allows to simulate the flow of air around a cavity.
- **Optimization:** It allows building an iterative algorithm to improve performance.





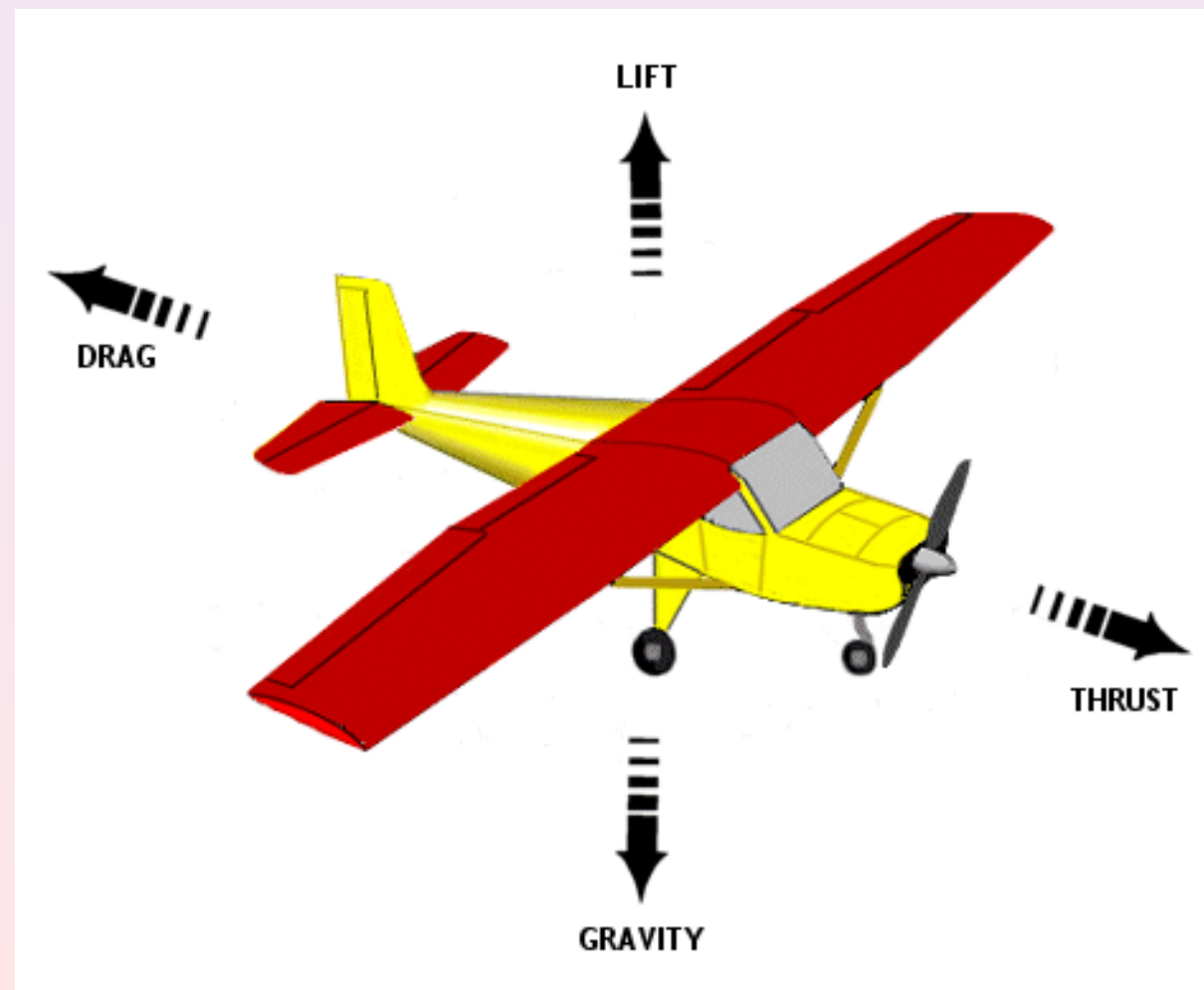
Computed pressure field over the surface of the airplane and flow lines of particles of air.

The method consists on formulating the problem in the context of the Calculus of Variations. To minimize

$$J(\Omega^*) = \min_{\Omega \in \mathcal{C}_{ad}} J(\Omega)$$

where \mathcal{C}_{ad} is the class of **admissible shapes** Ω , and $J =$ is the **cost functional** measuring the efficiency of the design (drag, lift,...)

J depends on Ω but not directly, rather thorough $u(\Omega)$, the solution of the air-dynamics in the exterior of the airplane.



Leonhard Euler

(1707-1783) derived the equations for the motion of perfect fluids, in the absence of viscosity:

$$u_t + u \cdot \nabla u = \nabla p.$$

But D'Alembert observed that the flight of birds would be impossible according to that model.

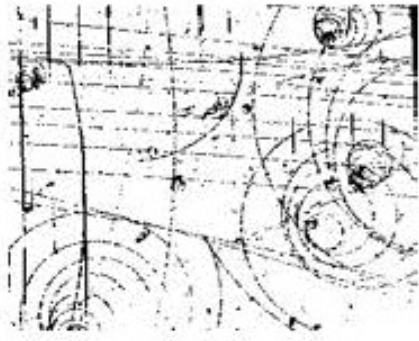
Claude Louis Marie Henri Navier (1785-1836) and Sir George Gabriel Stokes (1819-1903) much later incorporated the viscosity term:

$$u_t - \nu \Delta u + u \cdot \nabla u = \nabla p.$$

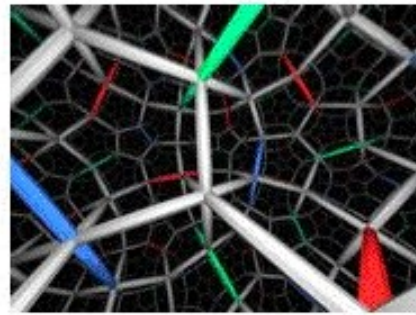
There are many open complex problems in the field of Fluid Mechanics.



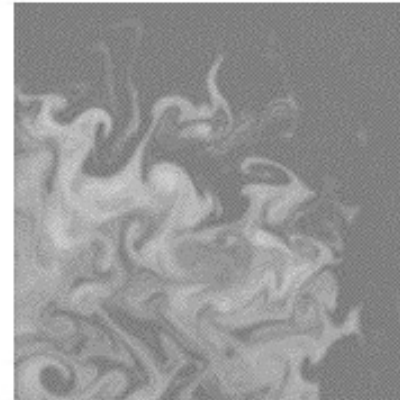
Fluid Mechanics is one of the most important areas of Physics because of its impact on our life : air, water, blood,...



Yang-Mills and Mass Gap



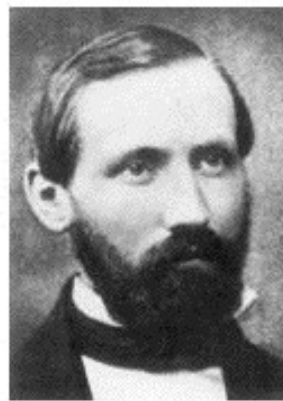
Poincaré Conjecture



Navier-Stokes Equation



Birch and Swinnerton-Dyer
Conjecture



Riemann Hypothesis



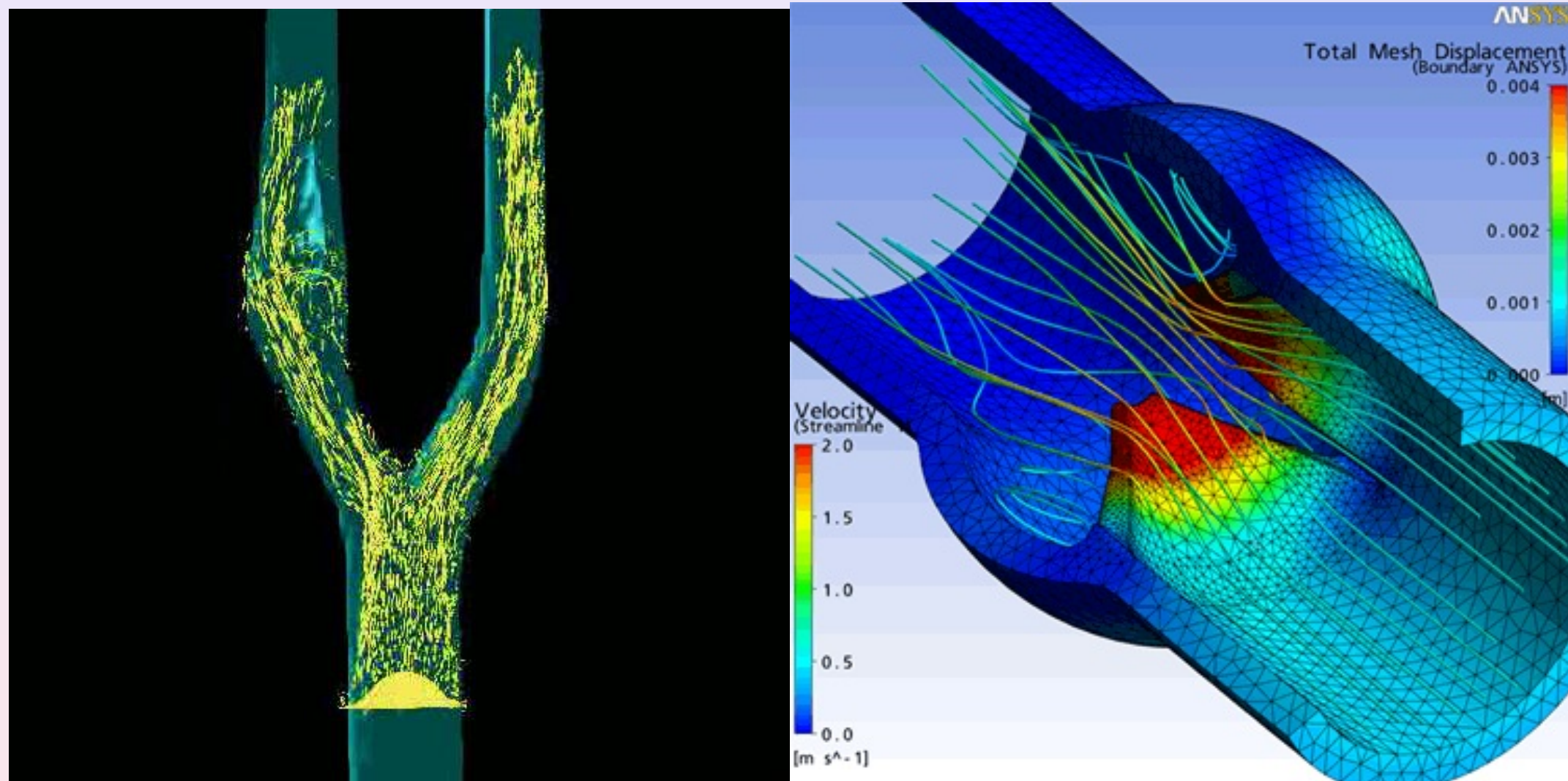
P vs NP Problem

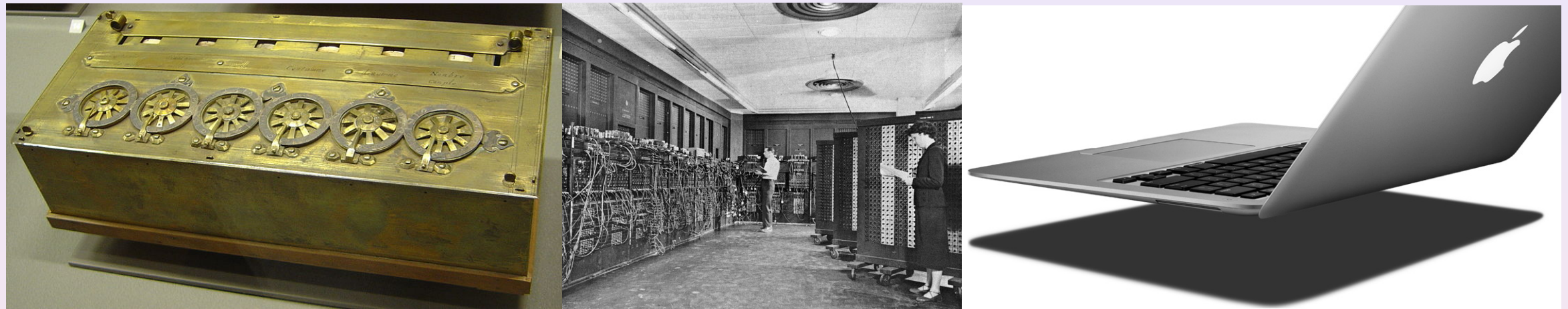


Hodge Conjecture

The millenium problems

The science program is still ongoing to a large extent thanks to computers.





[Pascaline](#), Blaise Pascal, 1645; [ENIAC](#): Electronic Numerical Integrator And Computer, 1946; [Macbook Air](#), 2008.

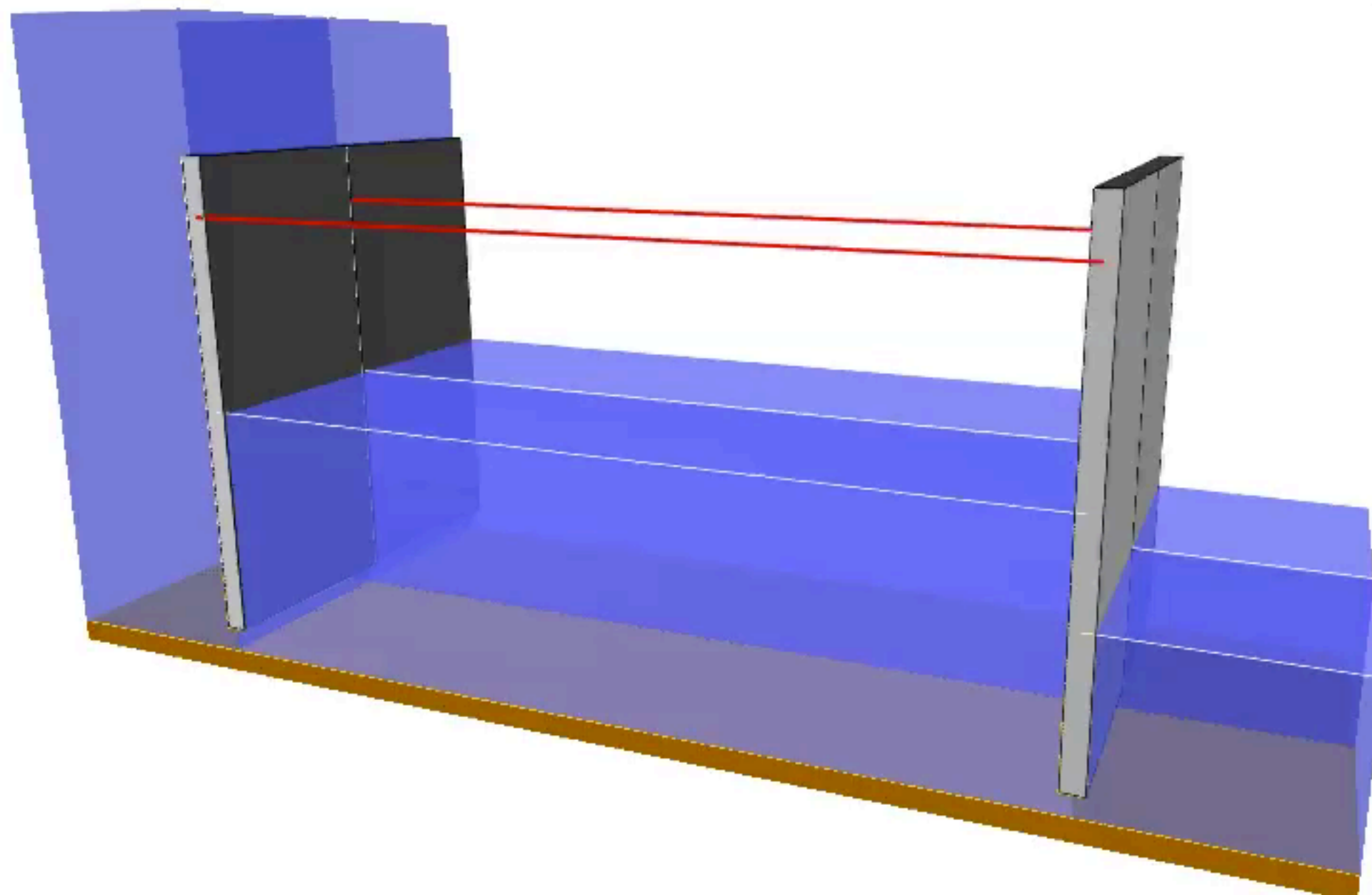


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- 5 Optimization**
- 6 Perspectives

An example in logistics

This is a typical and ubiquitous example in linear programming. A company s_i , $i \leq 1 \leq M$ items in each of the M storage locations. N clients request r_j items each, $1 \leq j \leq N$. The cost of transportation between the i -th storage location and the j -th client is c_{ij} . We have to decide about the number of items to be delivered from the i -th storage location the j -th client, v_{ij} .

Of course we want to minimize the cost of transportation. The problem is then that of minimizing the functional

$$\inf_{\{v_{ij}\}} \left(\sum_{i=1}^M \sum_{j=1}^N c_{ij} v_{ij} \right)$$

under the constraints

$$v_{ij} \geq 0; \sum_{j=1}^N v_{ij} \leq s_i; \sum_{i=1}^M v_{ij} = r_j, 1 \leq i \leq M; 1 \leq j \leq N.$$

In mathematics, computational science, or management science, mathematical optimization (alternatively, optimization or mathematical programming) refers to the selection of a best element from some set of available alternatives.

- Convex programming
- Linear programming
- Semidefinite programming
- Conic programming
- Stochastic programming
- Robust programming
- Combinatorial optimization
- Dynamic programming
- Heuristics and metaheuristics
-

These tools are so much used that nowadays there is plenty of software available both free and commercial: IPOPT

Last modified on 08/08/11 20:01:00

Welcome to the Ipop home page

Note that these project webpages are based on Wiki, which allows webusers to modify the content to correct typos, add information, or share their experience and tips with other users. You are welcome to contribute to these project webpages. To edit these pages or submit a ticket you must first [register and login](#).

Introduction

Ipop (Interior Point OPTimizer, pronounced eye-pea-Opt) is a software package for large-scale [nonlinear optimization](#). It is designed to find (local) solutions of mathematical optimization problems of the form

$$\begin{array}{ll} \min & f(x) \\ \text{s.t.} & g_L \leq g(x) \leq g_U \\ & x_L \leq x \leq x_U \end{array}$$

where $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ is the objective function, and $g(x): \mathbb{R}^n \rightarrow \mathbb{R}^m$ are the constraint functions. The vectors g_L and g_U denote the lower and upper bounds on the constraints, and the vectors x_L and x_U are the bounds on the variables x . The functions $f(x)$ and $g(x)$ can be nonlinear and nonconvex, but should be twice continuously differentiable. Note that equality constraints can be formulated in the above formulation by setting the corresponding components of g_L and g_U to the same value.

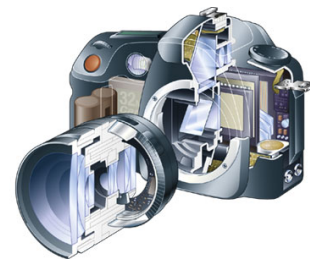
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Control theory and applications

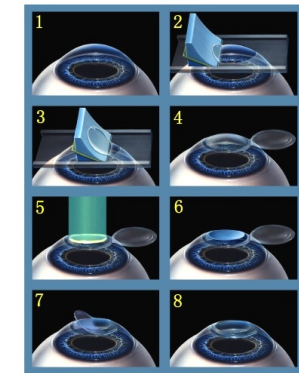
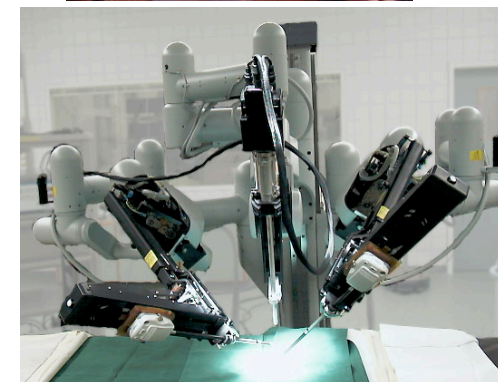
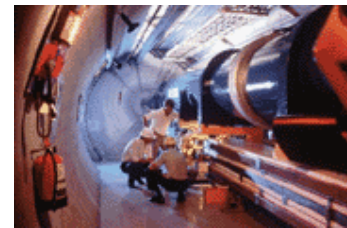
Mechanics

Vehicles (guidance, dampers, ABS, ESP, ...),
Aeronautics, aerospace (shuttle, satellites), robotics



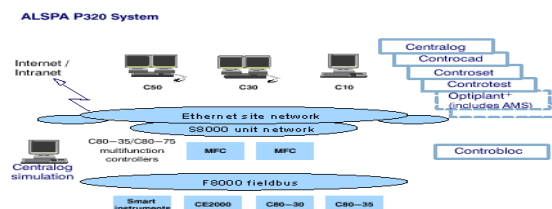
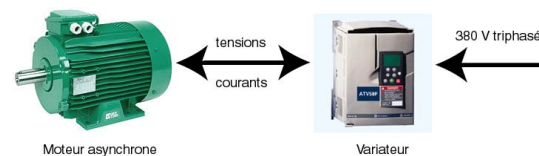
Biology, medicine

Predator-prey systems, bioreactors, epidemiology,
medicine (peacemakers, laser surgery)



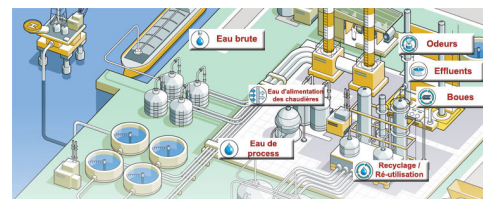
Electricity, electronics

RLC circuits, thermostats, regulation, refrigeration, computers, internet and telecommunications in general, photography and digital video



Chemistry

Chemical kinetics, engineering process, petroleum, distillation, petrochemical industry

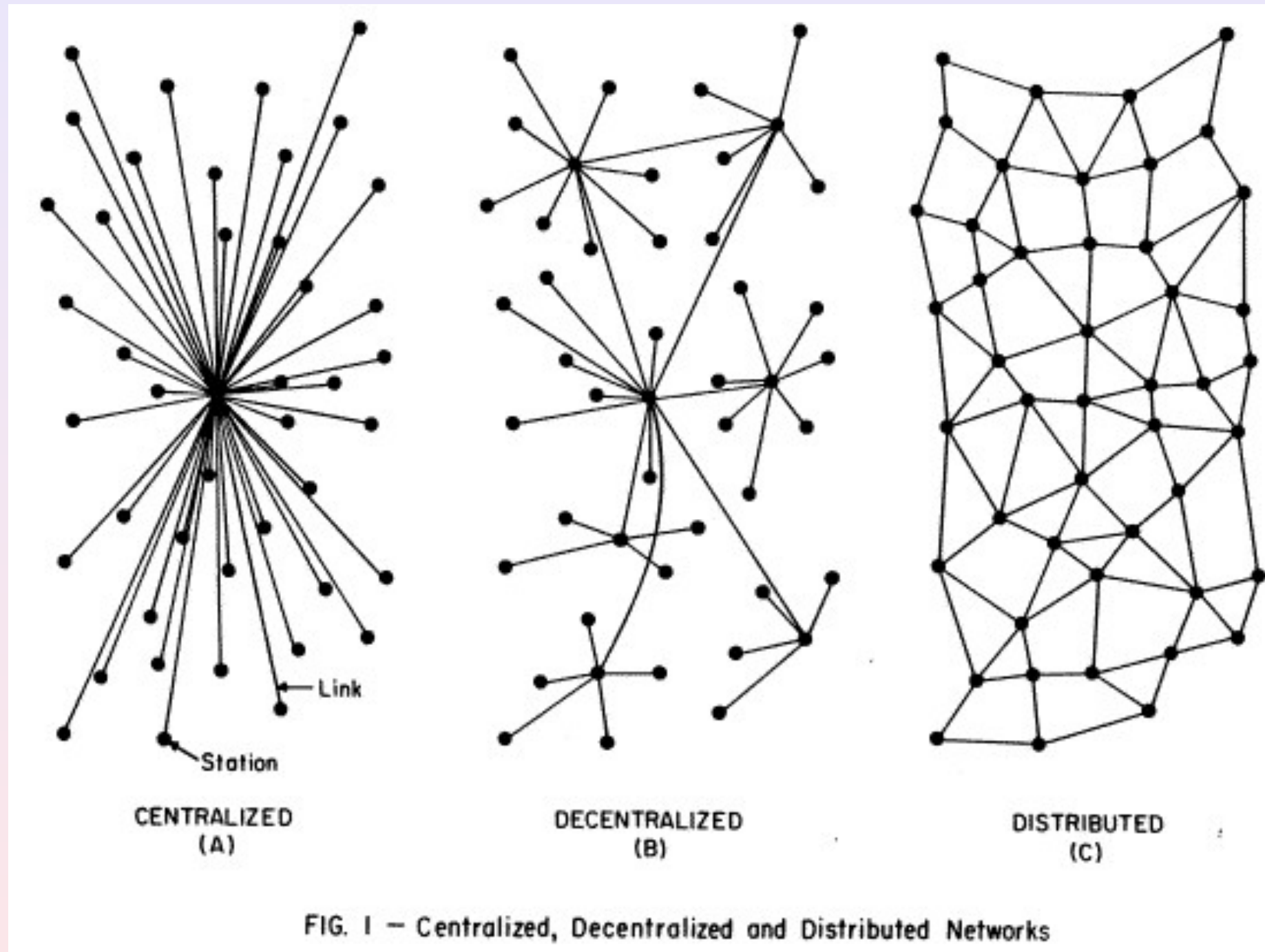


Economics

Gain optimization, control of financial flux,
Market prevision



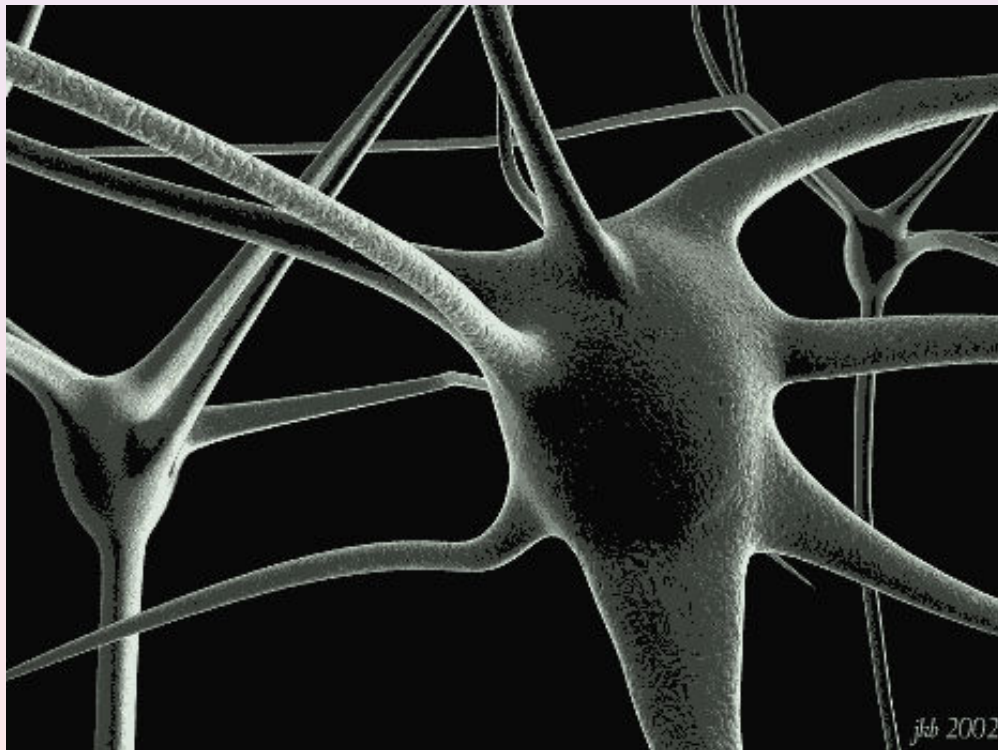
We live more and more on a complex network



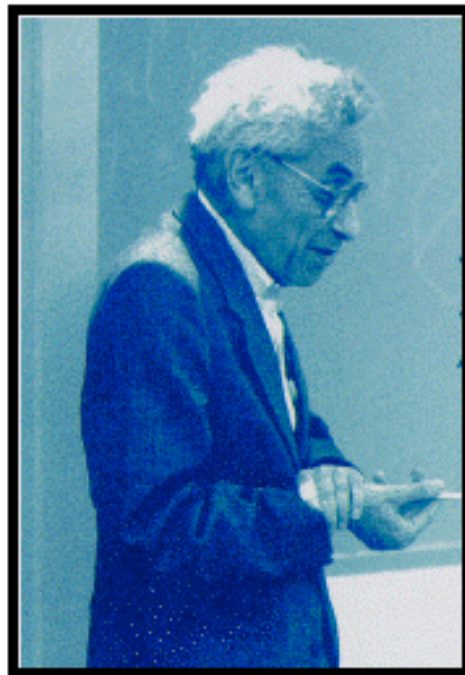
Mathematics are and will be increasingly influenced by the challenge of dealing with **complexity** and **multidisciplinarity**. The following areas will gain relevance:

- Discrete mathematics, combinatorics, graphs,...;
- Data mining/Big data;
- Statistical learning;

and other fields of research such as **neurosciences** and **social sciences**.



A mathematician is a machine for turning coffee into theorems



The Erdős Number Project

This is the website for the Erdős Number Project, which studies research collaboration among mathematicians.

The site is maintained by **Jerry Grossman** at **Oakland University**. **Patrick Ion**, a retired editor at **Mathematical Reviews**, and **Rodrigo De Castro** at the **Universidad Nacional de Colombia, Bogota** provided assistance in the past. Please address all comments, additions, and corrections to Jerry at grossman@oakland.edu.

Erdős numbers have been a part of the **folklore of mathematicians** throughout the world for many years. For an introduction to our project, a description of what Erdős numbers are, what they can be used for, who cares, and so on, choose the "What's It All About?" link below. To find out who **Paul Erdős** is, look at this **biography** at the MacTutor History of Mathematics Archive, or choose the "Information about Paul Erdős" link below. Some useful information can also be found in **this Wikipedia article**, which may or may not be totally accurate.

Paul Erdős (1913–1996) was a Hungarian mathematician. He published more papers than any other mathematician in history, working with hundreds of collaborators. He worked on problems in combinatorics, graph theory, number theory, classical analysis, approximation theory, set theory, and probability theory.

There will be unexpected advances in computing algorithms...

Year	Development	Key early figures
263	Gaussian elimination	Liu, Lagrange, Gauss, Jacobi
1671	Newton's method	Newton, Raphson, Simpson
1795	Least-squares fitting	Gauss, Legendre
1814	Gauss quadrature	Gauss, Jacobi, Christoffel, Stieltjes
1855	Adams ODE formulas	Euler, Adams, Bashforth
1895	Runge-Kutta ODE formulas	Runge, Heun, Kutta
1910	Finite differences for PDE	Richardson, Southwell, Courant, von Neumann, Lax
1936	Floating-point arithmetic	Torres y Quevedo, Zuse, Turing
1943	Finite elements for PDE	Courant, Feng, Argyris, Clough
1946	Splines	Schoenberg, de Casteljau, Bezier, de Boor
1947	Monte Carlo simulation	Ulam, von Neumann, Metropolis
1947	Simplex algorithm	Kantorovich, Dantzig
1952	Lanczos and CG iterations	Lanczos, Hestenes, Stiefel
1952	Stiff ODE solvers	Curtiss, Hirschfelder, Dahlquist, Gear
1954	Fortran	Backus
1958	Orthogonal linear algebra	Aitken, Givens, Householder, Wilkinson, Golub
1959	Quasi-Newton iterations	Davidon, Fletcher, Powell, Broyden
1961	QR algorithm for eigenvalues	Rutishauser, Kublanovskaya, Francis, Wilkinson
1965	Fast Fourier transform	Gauss, Cooley, Tukey, Sande
1971	Spectral methods for PDE	Chebyshev, Lanczos, Clenshaw, Orszag, Gottlieb
1971	Radial basis functions	Hardy, Askey, Duchon, Micchelli
1973	Multigrid iterations	Fedorenko, Bakhvalov, Brandt, Hackbusch
1976	EISPACK, LINPACK, LAPACK	Moler, Stewart, Smith, Dongarra, Demmel, Bai
1976	Nonsymmetric Krylov iterations	Vinsome, Saad, van der Vorst, Sorensen
1977	Preconditioned matrix iterations	van der Vorst, Meijerink
1977	MATLAB	Moler
1977	IEEE arithmetic	Kahan
1982	Wavelets	Morlet, Grossmann, Meyer, Daubechies
1984	interior-point methods	Fiacco, McCormick, Karmarkar, Megiddo
1987	Fast multipole method	Rokhlin, Greengard
1991	Automatic differentiation	Iri, Bischof, Carle, Griewank

Supervised learning

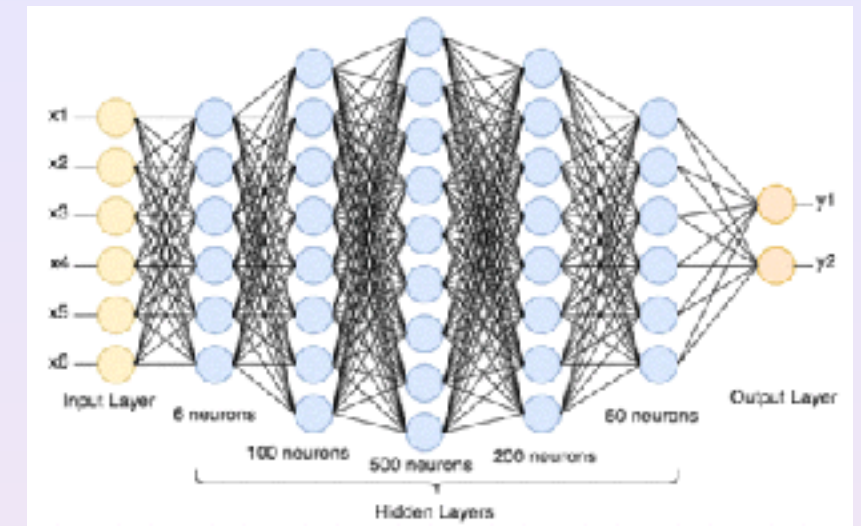
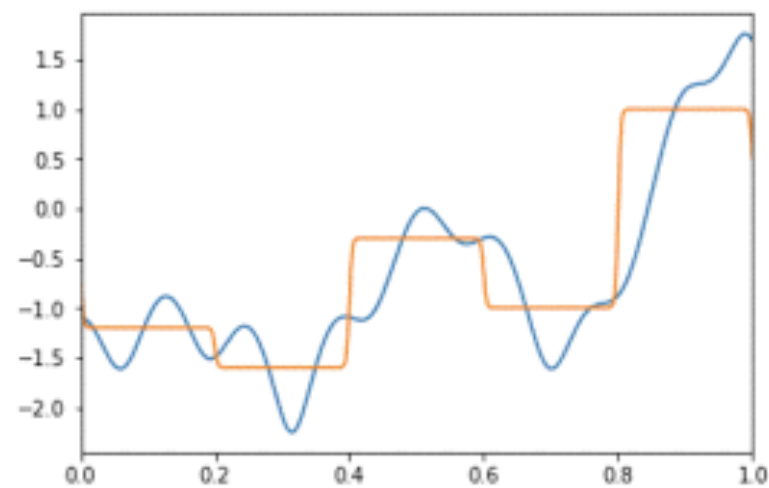
Goal: Find an approximation of a function $f_\rho : \mathbb{R}^d \rightarrow \mathbb{R}^m$ from a dataset

$$\{\vec{x}_i, \vec{y}_i\}_{i=1}^N \subset \mathbb{R}^{d \times N} \times \mathbb{R}^{m \times N}$$

drawn from an unknown probability measure ρ on $\mathbb{R}^d \times \mathbb{R}^m$.



Universal approximation theorem



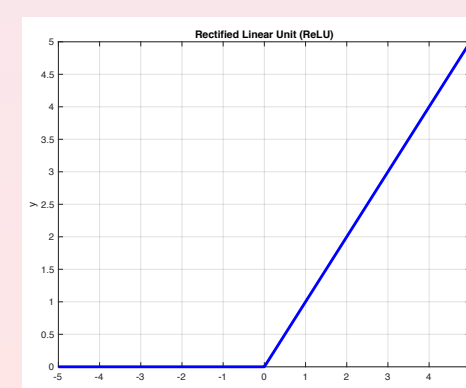
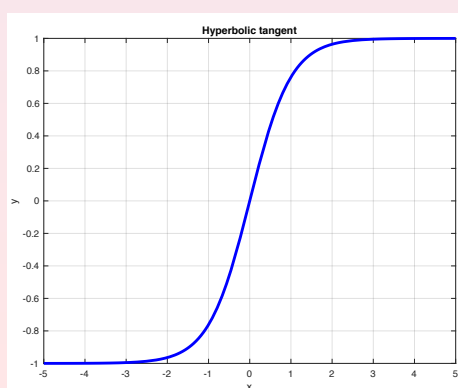
Math. Control Signals Systems (1989) 2: 303–314

**Mathematics of Control,
Signals, and Systems**

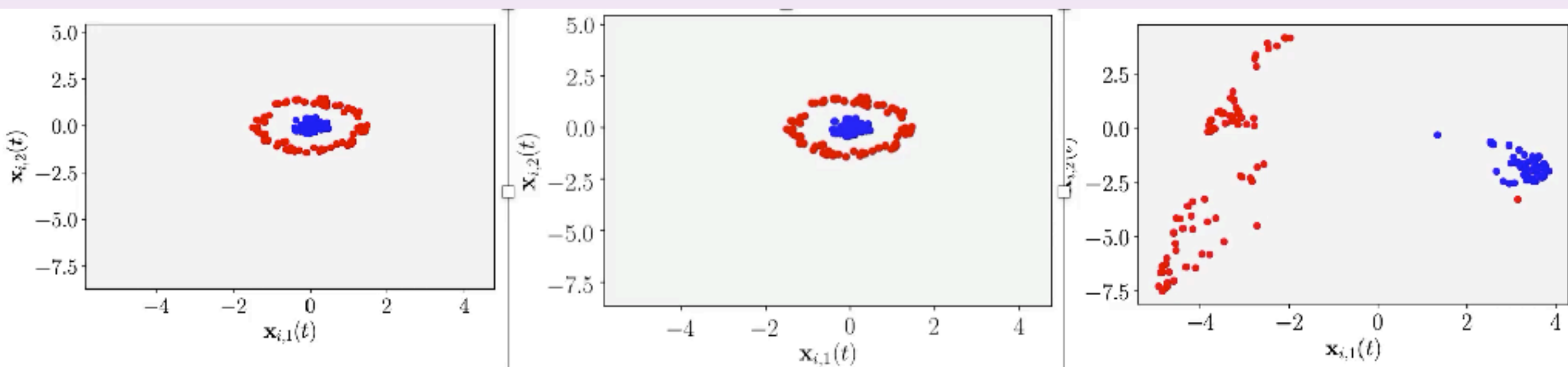
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Approximation by Superpositions of a Sigmoidal Function*

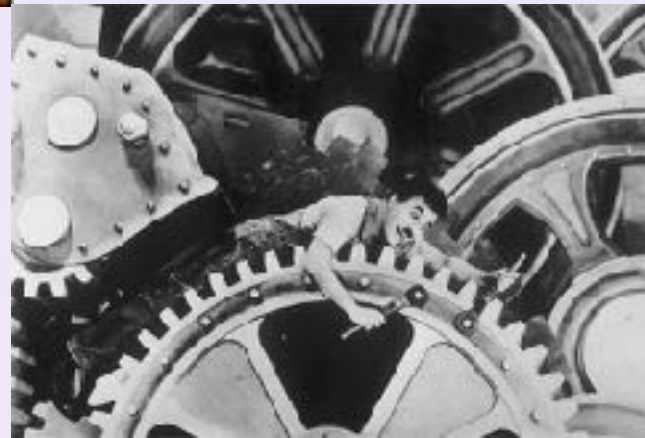
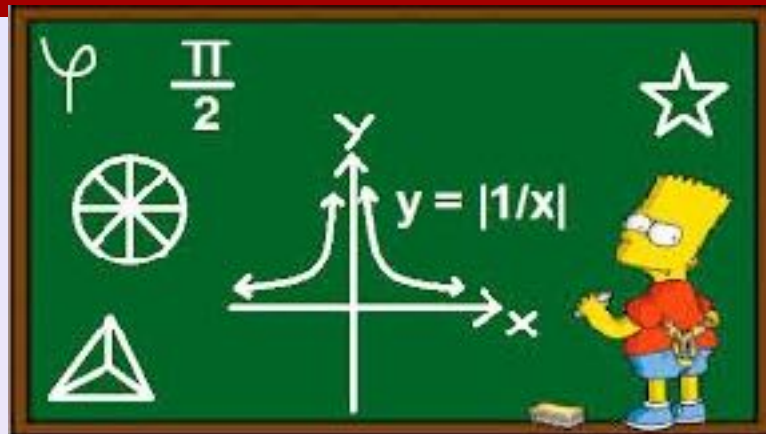
G. Cybenko†



$$T \rightarrow \infty \quad \sim \quad N_{\text{layers}} \rightarrow \infty$$







Bol. Soc. Esp. Mat. Apl. n°26(2003), 79–140

Control theory: History, mathematical achievements and perspectives*

E. FERNÁNDEZ-CARA¹ AND E. ZUAZUA²



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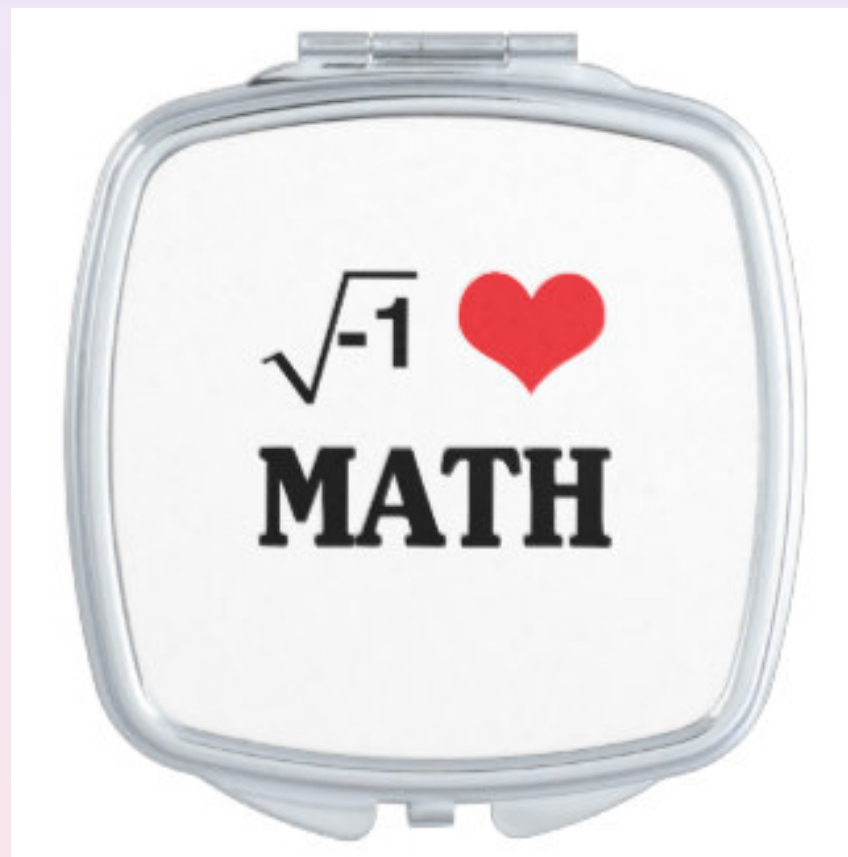


Early History of Machine Learning

Alexander L. Fradkov *

** Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, Saint-Petersburg (e-mail: Alexander.Fradkov@gmail.com)*

$$i = \sqrt{-1}$$



Enrique Zuazua

<https://www.caa-avh.nat.fau.eu>